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PROVISIONAL SPECIFICATION.

Improved Method and Apparatus for the Systematic Alteration or Distortion of Plane Pictures and Images by Means of Lenses and Mirrors for Photography and for other purposes.

I, THEODOR SCHEIMPFLUG, of 39 Sternwartestrasse, Vienna XVIII, Austria, Captain, do hereby declare the nature of this invention to be as follows: —

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Improved Method and Apparatus for the Systematic Alteration or Distortion of Plane Pictures and Images by Means of Lenses and Mirrors for Photography and for other purposes.

I, THEODOR SCHEIMPFLUG, of 39 Sternwartestrasse, Vienna XVIII, Austria, Captain, do hereby declare the nature of this invention to be as follows: —

It is well known in the practice of photography that in reproductions, slight alterations of the original picture can be readily effected without affecting the clearness of definition of the photographic picture by slightly inclining the original picture and the receiving plate relatively to the objective. Reproducing apparatus are also known which allow of utilizing this circumstance. But this alteration of the original picture has hitherto been done solely in an empirical manner and it is a very tedious and troublesome operation, which can be carried out only within very narrow limits and with a comparatively small degree of accuracy. For this reason it has often been preferred to attempt to obtain the alteration of the original picture without reference to the clearness of definition of the photographic picture, and to obtain the requisite clearness of definition of the photographic picture by the use of very small screens and strong light.

But on the other hand it is a matter both of scientific as well as of commercial interest to be able to carry out such alterations of the original picture in an exact and systematic manner, and the present invention has for its object to provide an improved method based on exact and mathematical principles, of solving this problem, and also improved apparatus for carrying out the said improved method.

In the accompanying drawings, Figs. 1 and 2 are respectively a section and a perspective. view illustrating the meaning of the terms "projective" and "perspective", "axis of collineation" and "axis of projection".

Figs. 3 and 4 illustrate in perspective view and in section the meaning of the terms "counter plane" and "counter axis", and the consequent geometrical relations of the two collinear pictures to each other.

Fig. 5 illustrates the path which is described by conditions oblique reproduction, so far as they are known.

Fig. 6 illustrates the mutual perspective relations of several projective images, the planes of which intersect one another in the same straight line.

Figs. 7, 8 and 9 illustrate the condition of affinity; that is to say:—

Fig. 7 illustrates the case in which all the images remain upright;

Fig. 8 illustrates the case in which there is first an inversion of the image and then the image is re-erected, and Fig. 9 illustrates the case in which first an erect image is produced which is then inverted, and *vice versa.*

Figs. 10 and 11 illustrate the optical conditions for the production of images

in planes inclined to the optical axis with the use of spherical lenses, and the connection between the geometrical relations and the optical relations of the picture and its image thus produced, said connection constituting the basis of the improved method;

Fig. 10 illustrates the use of a symmetrical objective, in the case where the two principal planes coincide with the plane of the objective. Fig. 11 illustrates the case of an unsymmetrical objective, and more particularly of a tele-objective, where the distance between the two principal planes can no longer be neglected.

Figs. 12 and 13 illustrate the laws of the production of images when spherical mirrors are employed.

Figs. 14 and 15 illustrate the procedure which forms the basis of the improved method for the production of affinite pictures by optical means, that is to say, Fig. 14 illustrates the case where the positions of the original and of the photographic picture are convertible; and Fig. 15 illustrates a case where the original can be inserted on one side only.

Fig. 16 illustrates a way of carrying out the improved method for the purpose of direct production of affinite alterations of the original picture.

The remaining figures illustrate various constructional forms of apparatus adapted to carry out the improved method under various given conditions.

Figs. 17, 17^a, $17^b$and 17^c are respectively a side elevation, a plan and two detailed views of what is known as a slide apparatus for projective alterations of image. Figures 18 and 18^a serve to illustrate the principle of the positive actuating mechanism employed in the said apparatus. Fig. 19 is a diagram, and Fig. 19^a is a sectional view of a practical construction of apparatus designed for the production of alterations of image only within certain limits, but with great accuracy. Fig. 19^b illustrates as an appurtenant detail a turn-table which is also employed in all the other apparatus. Figs. 20, 21 and 22 illustrate diagrammatically modifications of the apparatus shown in Fig. 19.

Figs. 23, 23^a and 23^b are respectively a plan, a side elevation, and a section of a constructional form of apparatus which secures the permanent fulfilling of the second condition of clearness of definition of image and which has resulted from the original form shown in Fig. 19. Figs. 24, 24^a, and 24^b, are respectively a plan, a horizontal section and a vertical section of a second constructional form of apparatus fulfilling this condition and arranged in the form of a book. Figs. 25 and 25^a are diagrammatic representations of an apparatus which is adapted to produce affinite alterations of image without the production of an intermediate image, and in which only mirrors are employed. Fig. 26 shows likewise diagrammatically an apparatus which produces in a direct manner affinite alterations of image and in which a mirror is combined with an objective. Figs. 27, 27^a and 27^b illustrate in like manner an apparatus for directly affinite alteration of image in which only mirrors are employed.

Figs. 28 and 28^a illustrate an apparatus for directly affinite alterations of image in which only lenses are employed.

In accordance with the newer geometry or geometry of position, two plane figures *a, b, c* . . . . and a^1, b^1, c^1, (Figs. 1 and 2) which have been produced by the intersection of the same bundle of rays by two planes A and B, are described as being "projective" or "collinear" to each other, and every relative position of those planes in which all the straight lines connecting two points $a\ a^1$, $b\ b^1$, $c\ c^1$ of the respective planes, meet in one point *o,* is called a perspective position. The point *o* from which these connecting straight lines (lines of sight) diverge, is called the centre of projection or of collineation, and the straight line of intersection M M of the two planes A and B which is common to the two figures under consideration, is called the axis of collineation. Since the axis of collineation is common to the two figures, every straight line of the original will intersect its image in a point in the axis of collineation and *vice versa.*

Again, the planes A^1 and B^1 (Figs. 3 and 4) drawn through the centre of projection *o,* parallel to the two planes A and B, are called counter planes, and they intersect the planes of the pictures in the images of the infinitely distant straight lines of the planes B and A. These lines, g^1 and g^2 are called counter-axes and are the geometrical loci of the points of intersection of all the mutually intersecting straight lines which appear as parallel straight lines in the plane of the other picture. That is to say the counter-axis g^1 situated in the plane A is the geometrical locus of the points of intersection of all the straight lines which appear as parallel straight lines in the plane B, and *vice versa.*

An affinite alteration of an original, that is to say, an alteration such that the image is lengthened or shortened in a determined direction, but remains completely unaltered in another direction, would be possible in a direct manner, such as by the incidence of shadow, only when the centre of projection is situated at an infinite distance. That is to say, plane pictures on the same scale which have an affinite relationship to one another, may first be conceived as figures of intersection of two planes with one and the same cylinder of rays. But the geometry of position teaches that yet another conception is possible. If according to Fig. 6, a plane picture in the plane M N be projected from the point o^1 on to the plane M Q and also from the point o^{11} on to the plane M R-, and the three planes M N, M Q, and M R intersect one another in one and the same straight line M M, which is the common axis of collineation of the three pictures, then the two images projected on the planes M Q and M R will be situated in perspective to each other, and their common centre of collineation o^{111} will be situated in the straight line $o^1 o^{11}$. This new centre of collineation o^{111} may however be also situated at an infinite distance. When this is the case the pictures on M Q and M R are in affinite relationship to each other (Fig. 7). Affinity is merely a particular case of projection. However, notwithstanding this fact, for the sake of brevity, the term "projective" will hereinafter be employed, as opposed to "affinite", in the sense that the geometrical relations which are being considered herein, will always be termed "projective" when the centre of projection o^{111} is situated at a finite distance, and will be termed "affinite" when the centre of projection o^{111} is situated at an infinite distance.

The following axioms which combine optics and the new geometry, constitute the theoretical basis of my improved method and of my improved apparatus for carrying out the same.

1. Photographic images are perspective images in a strictly mathematics sense, and the principal (nodal) points of the objective that is employed; serve as the centre of projection of their corresponding images; that is a well known fundamental fact of "photogrammetry".

2. Every plane image is reproduced as a plane image by systems of spherical lenses or mirrors arranged in any combination, within the limits of accuracy to which the general lens equation or mirror equation is applicable. This is also a well known fact.

3. In oblique reproduction it is well known that the plane of the original and the first principal plane of the system of lenses, as also the plane of the projection image and the second principal plane of the system of lenses intersect each other in optically conjugate straight lines (Fig. 5).

If the two nodal points $h_1 h_{11}$ combine to form the optical centre O_1 whereby the two principal planes are caused to coincide with the objective plane M O, the two image planes and the objective plane intersect in the same straight line M which serves as the geometrical axis of collineation of the two images

The general lens equation assumes the form: —

$$\tan e = \frac{F}{b_0 - F} \tan i$$

where *e* and *i* are the angles enclosed by the image planes and the two principal planes or the objective plane. F is the focus of the objective namely,

+ F for converging lenses
- F for diverging lenses.

b_0 is the distance of the point of intersection of the optical axis of the objective with the image plane A of the original from the first nodal point h_1 of the objective.

In this connection it should be noted that in most of the systems of lenses in practical use which are symmetrical as a rule, the two nodal points and therefore also the principal plane are situated so close to each other that they can be considered practically as coincident. Then it is allowable to speak of the "optical centre" and of the "plane of the objective" (Fig. 10). Only the tele-systems, that is to say, the combination of positive and negative lenses form an important exception to this rule, by reason of their considerable asymmetry, and in these systems, the nodal points or principal planes lie far outside of the system of lenses, namely on the side of the positive lens, whilst the two nodal points or principal planes are shifted far apart at the same time (Fig. 11).

Now the foregoing lens equation has a geometrical significance which is very important for the improved method. Namely, if the counter axes g_1g_2 of the two perspective images *a, b, c,* a^1, b^1, c^1, are determined by placing through the two nodal points h^1, h^{11} of the objective, planes parallel to the two image planes A and B (Fig. 10), and causing the former planes to intersect the image planes, then these counter axes are always situated in the focal planes $F_1F_{11}$of the lens system. The counter axes have thus not only a geometrical significance, but also an optical significance.

Further, the following axiom from the geometry of position, is of importance for the production of affinite alterations of image, namely that two optically perspective systems of images which are required to complement each other, must have the intermediate image and the appurtenant counter axis in common (Figs. 7, 8 and 9).

In the case of spherical mirrors (Figs. 12 and 13,) the two image planes A and B and the vertex plane S of the mirror intersect one another in the same straight line which fills the function of the geometric axis of collineation of the two images.

The general equation for a mirror (vertex equation) is as follows: —

$$\begin{matrix} + \\ - \end{matrix} \frac{1}{F} = \frac{1}{R.\tan e} + \frac{1}{R.\tan i}; \text{ if } R = M\,S, \text{ and } F = \frac{r}{2} =$$

half the radius of the curvature.

+ F refers to concave mirrors. -F refers to convex mirrors.

This mirror equation has the analogous geometrical signification as hereinbefore set forth: —

Namely, if the counter axes g_1g_2 of the perspective system of images be determined by drawing planes through the centre of curvature C of the mirror parallel to the two image planes A and B, and causing them to intersect the said two image planes, then these counter axes will always be situated in the focal plane F of the mirror.

It is upon the combination of these laws (part of which were known before, but the greater part of which are new) that the method of altering images as hereinafter described, as well as the apparatus relating thereto are based.

In carrying out the method, of producing projective alterations of images the procedure is as follows: —

After being determined with the help of the laws and principles hereinbefore set forth by mathematical or by graphic calculation, the necessary data for adjusting the original, the objective, and the image, then when the tele-

systems are employed as the objective, the first plane A containing the original, and the second plane B containing the dulled glass screen intended to receive the projected image, together with the corresponding (first or second) principal plane H_1H_{11} of the objective, are caused to intersect one another in such a manner that the two straight lines of intersection thereby produced will become optically conjugate straight lines in accordance with the laws of optics. In this manner the first condition for the clearness of definition of the image is fulfilled. Then one plane, say for instance the plane containing the dulled glass screen is rotated relatively to the other plane which in this case contains the original and which is kept stationary, until it is parallel to the counter plane which appertains to the aforesaid stationary plane and which is determined by the latter and by the focal plane (appertaining to it) of the objective. In this manner the second condition for the clearness of definition of the image is fulfilled, namely that the counter axes shall lie in the focal planes. Then an image of the original having the desired degree of distortion and with perfect clearness of definition, will appear on the dulled glass screen.

When symmetrical systems of lenses are used as an objective, in which the two principal planes thereof become the objective plane, the procedure is exactly the same, because the two planes that contain the original and the dulled glass screen are caused to intersect the objective plane in the same straight line and then one plane is rotated relatively to the other stationary plane until it is parallel to the counter plane of the said stationary plane.

When spherical mirrors are used, the foregoing also holds good as in symmetrical objectives, except that in the case of spherical mirrors the vertex plane of the mirror replaces the objective plane.

When affinite alterations of an image are to be produced, the procedure may be as follows: –The devices mentioned hereinbefore are arranged two or even more in number, one behind the other, so that the result of the first device, namely the intermediate image, is either first fixed by photography and then afterwards altered, or it may be immediately further altered by the second device without being so fixed. The same applies also to the result of this second device and its alteration in using the third device; and so on.

Figs. 14, and 15 serve to explain more fully the first method of affinite alteration of an image.

As shown in Fig. 14, the image plane Z of the intermediate image $a^{11}b^{11}c^{11}$, or the dulled glass screen of the first picture is kept stationary at any desired angle (i) relatively to the optical axis of the objective and the carrier of the intermediate image is shifted parallel to the objective plane towards the axis of collineation M or away from the same, according as it is desired to produce a shortening or a lengthening of the proportions of the image. Then the plane of the original is adjusted in such a manner as to cause the two planes and the objective plane to intersect one another in a straight line, and to bring the counter axis g^1 of the original within the focal plane. As soon as the image is clearly visible onthe dulled glass screen, the first picture is taken and fixed by photography; then the negative thus obtained is placed in the apparatus in the place from which it was taken. If this negative were now projected back into the plane of the original without any further adjustments, the result will be an exact reproduction of the original. But if the negative be shifted without altering the angle-of inclination i to the objective Plane (mirror plane) and without altering the distance from the latter, or if the objective (the mirror) be shifted in its plane without altering the position of the intermediate image (negative), then on projecting back affinite alterations of pictures can be produced in the most varied stages of alteration, after having fulfilled the conditions required for the production of a clearly defined image, by rotating the plane in which the original was originally situated, and on to which the affinitely altered image is being projected back.

This method presumes that the picture to be reproduced and the dulled glass

screen, may exchange places relatively to the objective. If this is not the case and if the negative must be fastened to the carrier of the original in order to carry out the second alteration, then the method undergoes a modification. The intermediate image (negative) $a^{11}b^{11}c^{11}$ is then, as shown in Fig. 15, arranged by the side of the original in such a manner that the counter axis g^1 of the original which was determined at the first "taking" as the straight line of the intersection of the plane Z with the focal plane (F_{11}), is now caused by the new adjustment to come within the focal plane (F_1). The shifting of the parts parallelly to the objective plane and the exact adjustment are effected in the same manner as in the preceding case.

But in the present case every alteration in the angle of inclination (i) of the intermediate image relatively to the objective plane, causes an alteration in the scale of the affinite alteration.

As above stated, the method for the production of complicated projective and affinite alterations of an image may also be carried out without taking a photograph of the intermediate image, by immediate further alteration of the same, and the necessary adjustments may be effected by correspondingly inclining or rotating two or more optical systems.

Direct affinite alterations of images may likewise be effected, for example, without taking a photograph of an intermediate image, by immediate further alteration of the latter, by collimating two optical systems (two lenses, one lens and one mirror, or two mirrors) with each other, that is to say, causing their focal planes to coincide with each other.

Two such systems collimated with each other, are unable to reproduce oblique pictures otherwise than as affinite pictures, because the plane of the intermediate image can intersect the common focal plane of the two optical systems in a single straight line only, that is to say, in the common counter axis, thus by itself fulfilling the condition for affinity.

Fig. 16 illustrates the carrying out of this method with the use of two hollow mirrors S and S^1. The original $a\ b$ is set within the field of vision of the two mirrors at any desired angle to the focal plane. The mirror S by reflecting the light rays from the original on to the mirror S^1, produces an imaginary or virtual intermediate image situated outside of the surface of the figure and which is transformed by the mirror S^1 into the affinitely altered image a^1b^1.

The practical carrying out of the method of projective alteration of an image is considerably facilitated by the use of apparatus which automatically give a constant clearness of definition of image, because then instead of the preliminary calculations and constructions, only the following simple rules need to be observed: —

1. Rotation of the original serves to place the straight lines which are to remain parallel in the original as well as in the distorted image, parallel to the axis of collineation, that is to say, it serves to place the axis of collineation of the original parallel to the axis of distortion of the apparatus, and then by shifting the image to bring it into coincidence with the axis of distortion.

2. Parallel straight lines in the original appear in the transformation as straight lines intersecting one another, and *vice versa.* The geometrical loci of the points of intersection are the counter axes.

3. The degree of the divergence or of the convergence of mutually intersecting images of parallel straight lines of the original and *vice versa,* is solely a function of the distance of the counter axes of the respective images from the axis of collineation. If this distance is smaller, then the divergence will be greater, when the distance is greater the divergence will be less.

4. The distance between the images of straight lines which are parallel to the axis of collineation, is in the transformation, smaller or larger than in the original, that is to say, the transformation appears to be shortened or lengthened in comparison with the original when it is shifted nearer to or further away from the axis of collineation than the original.

5. A rectangle in the original appears in the image as a trapeze when it is symmetrical to the optical axis of the objective; and it appears as a trapezoid when it is unsymmetrical and the angles of the trapezoid are the more dissimilar, the greater the unsymmetry of the position of the trapezoid.

6. The distance of the counter axis of the transformation from the axis of collineation which distance is the determining factor for the divergence or the convergence of the projective images of parallel straight lines in the original, is best regulated continuously by rotating the plane containing the transformed image, but it may also be altered intermittently by changing the objective, because it is also dependent on the focal length of the objective.

I shall now proceed to describe some constructional forms of apparatus adapted to carry out the improved method hereinbefore set forth.

The first step in the provision of practical apparatus was the adaptation of photographic reproducing apparatus in such a manner that the plane of the original and the plane of its image can be inclined to the optical axis of the objective. But in apparatus of this kind the clearness of definition of the image is the sole criterion of the proper adjustment. Consequently such apparatus are incomplete, inconvenient, and not capable of accurate work. An essential and industrially new improvement on these known constructions consists in the provision of sighting devices in the planes of the original and its image which allow of controlling the exact intersection of the two planes with the objective plane in the same straight line, that is to say, of satisfying the first condition of clearness of definition. Another way of attaining the same object but which entails a special modification of ordinary photographic reproducing apparatus when employed for these purposes and which constitutes a great constructional improvement consists in combining the holders or supports of the objective and of the original and its image constructionally in such a manner that they will always intersect one another in a straight line. Further by taking care that the second condition for clearness of definition of the image is satisfied automatically, then types of apparatus are evolved which render all preparatory calculation unnecessary in the case of projective alterations of image because the clearness of definition of the image is already constantly provided for, and as the projective alterations of the image in such apparatus take place according to very simple and clear laws, and as they are the only matter which the operator has to consider they may readily be carried out in a purely empiric manner. The apparatus which satisfy automatically the two conditions of clearness of definition of the image are therefore industrially the most valuable part of the present invention. But for the most industrial purposes projective alterations of image are generally of less importance than affinite alterations of image. In the latter case apparatus are important which enable affinite alterations of image to be effected without any intermediate process. These have been produced constructively by the combination or the arrangement in series of two apparatus for projective alterations in which the intermediate image (which together with its counter axis is common to both projective systems) remains merely an aerial or imaginary image.

The apparatus shown in Fig. 17 in side elevation and in Fig. 17[a] in plan consists of a photographic reproducing apparatus which is provided with all the devices that are necessary to carry out projective alterations of image in a systematic manner. For this purpose the frames *r s* and *v w* which are designed to receive the original and the plate holder for the image are arranged to rotate on vertical axes $x\ x^1$ and $y\ y^1$ that are mounted in the supports *f g* and *h k*. These supports are capable of being shifted rectilinearly and perpendicularly to the axis of the objective.

The shifting of the supports *f g* and *h k* parallel to the plane of the objective and in opposite directions at the same time, takes place along the straight guides T T and is effected by the lever E which is pivoted on the pin e^1 and is

actuated by means of a screwed spindle I and a nut K. The rotary motions of the plane of the original and the plane of the image relatively to each other and to the plane of the objective are effected by means of a positive actuating mechanism in such a manner that both the conditions of clearness of definition of the image are satisfied, and therefore when the apparatus has been once rectified, it will produce permanently clearly defined images.

The said positive actuating mechanism consists of a screwed. spindle J which is arranged in the objective plane and drives a nut K^1 carrying two pivoting guides L in which can slide arms P rigidly connected to sleeves S mounted loose on the parallel rotary axes $x\ x^1$ and $y\ y^1$ of the supports $r\ s$ and $v\ w$.

The rotary motion of the sleeves S is transmitted by means of suitable mechanism (such as for instance bevel wheel gearing G) in the same amount but in the opposite direction of rotation to the rotary frames $r\ s$ and $v\ w$.

The mode of operation of this positive actuating mechanism is illustrated by Fig. 18.

The first condition to be satisfied in order to produce clearness of definition of the image is the following:—The plane of the original, the plane of the image, and the plane of the objective must, always intersect one another in a straight line M.

As in the great majority of cases only slight inclinations of the plane of the original and the plane of the image are required, this straight line M is situated as a rule far outside the apparatus and it is therefore difficult to utilize it for the purposes of construction. Consequently, it is necessary to substitute for this straight line M another but more easily followed mark m.

If, as shown in Fig. 18, the distances of the axes of rotation $x\ x^1$ and $y\ y^1$ from the plane of the objective M O are made equal to each other and to twice the focal length of the objective employed, and if

$$\text{the angle } Ox_2m^1 = \text{angle } O\ M\ x = e$$
$$\text{angle } Oym = \text{angle } Omy = i$$

then the triangle O x M is similar to the triangle O m x. The triangle O y M is similar to the triangle O m y, because all their angles are equal; therefore

$$O\ M.Om^1 = Ox^2$$
$$O\ M.Om^2 = Oy^2$$

But as it was assumed that O x = O y, then $O\ m^1 = O\ m^2 = Om$; that is to say m^1 and m^2 meet at the point m. Now this point. m is very convenient for use for the purposes of construction, and its place is taken by the nut K^1. The rotary arms P take the place of the lines $m\ x$ and $m\ y$, and the supports $r\ s$ and $v\ w$ take the place of the lines M x and M y. The bevel wheel, gearing or other mechanism employed makes the angles of rotation e and i of the arms P and of the supports $r\ s$ and $v\ w$ equal to each other and opposite in direction. Further the arms P by reason of their pivoting guides L meet always at the point m of the plane of the objective. Consequently the picture planes of the supports $r\ s$ and $v\ w$ will be caused to intersect each other always in the straight line M in the plane of the objective.

But since O x O y; then the counter planes O g_1 and O g_2 divide the triangle x M y into the two similar and equal triangles x g_1O and O g_2y and the parallelogram O g_1M g_2; and

$$x\ g_1 = g_1M, \text{ and}$$
$$y\ g_2 = g_2M.$$

The straight lines g_1 and g_2 are situated at half the distance of the straight lines $x\ x^1$ and $y\ y^1$ from the plane of the objective, and they lie therefore in the focal planes.

In this manner the two conditions for producing clearness of definition of the images are strictly satisfied.

Fig. 18[a] illustrates a modification of the positive actuating mechanism just described.

In this case the distance of the axes of rotation $x\,x^1$ and $y\,y^1$ from the plane of the objective are variable, but they must also in this modification be capable of being moved to and fro in straight, guides parallelly to the plane of the objective.

If as shown in Fig. 18[a], –

angle O x m^1= angle OM y = i

angle 0 y m^2=angle OMx=e

then triangle O x M is similar to triangle O y m^2 and triangle O y M is similar to triangle O x m^1, therefore $\frac{O\,M}{O\,x} \cdot \frac{Ox_1}{Om} = \frac{\sin a \,.\, \sin b}{\sin e \,.\, \sin i} = \frac{O\,M}{O\,y} \cdot \frac{O\,y_2}{O\,m}$. Consequently O m^1= O m^2= O m, that is to say m^1 and m^2 meet at the point m.

From this it follows that: If the point m be determined in the manner shown in Fig. 18[a], namely by drawing the angle e at y and the angle i at x on the opposite side of M, then the distances of the axes of rotation $x\,x^1$ and $y\,y^1$ need no longer be equal to each other and to twice the focal length of the objective employed.

But the positive actuating mechanism requires to be arranged in a somewhat different manner: —A nut K^1 with pivoting guides moves along the plane of the objective. The arms P sliding in the guides of the nut K^1, rotate the sleeves S which are mounted loose on the axes $x\,x^1$ and $y\,y^1$. But the rotary movements of these loose sleeves S are now no longer transmitted to the supports for the original and image mounted on the same axis, but are transmitted by means of suitable mechanism (cord gearing, parallelo guides &c.) to the oppositely situated supports for the original and image in equal angles but in opposite directions.

With this arrangement the clearness of definition is not absolutely ensured as in the arrangement before described, and it is maintained only when it has been once obtained. Namely, in Fig. 18[a], the counter axes g_1 and g_2 will move strictly parallel to the plane of the objective, so long as x and y are likewise moved parallel to the plane of the objective. Consequently, when these counter axes have been set in the focal planes, by having once obtained the clearness of definition of the image, they will remain in those planes, and the clearness of definition of the image will likewise be maintained.

For the purpose of also satisfying the highest requirements, and of facilitating an exact rectification of the apparatus, the following arrangement is further provided.

Two slit-apertures lamps p, which are respectively keyed fast on the vertical axles of rotation $x\,x^1$ and $y\,y^1$ of the supports $r\,s$ and $v\,w$, throw beams of light constantly in the direction of the planes of the original and of the image. When all the parts are in the middle position, those beams of light must coincide with the sights provided on the supports $f\,g$ and $h\,k$, and when the supports $r\,s$ and $v\,w$ are reclined sufficiently, they must meet in a straight line M on the screen u situated in the plane of the objective.

In cases where the requisite inclinations of the planes of the original and image are so great that the positive actuating mechanism no longer suffices, the positive actuating mechanism may be thrown out of engagement, and then the beams of light will serve also as sighting and adjusting means for causing the two planes to intersect the plane of the objective always in the straight line M.

Certain devices are also provided for the purpose of rendering the apparatus capable of solving even the more difficult problems which can be controlled by mathematical calculation. Such devices are: —

Division-marks which indicate the respective distances of the vertical axes

of rotation $x\ x^1$ and $y\ y^1$ from the optical axis of the objective; divided circles which allow of reading off the angles e and i through which the supports $r\ s$ and $v\ w$ are rotated; a turn-table U constructed in the support $r\ s$ so as to allow, when inserting a picture into the apparatus, of setting the axis of collineation or the counter axis of the picture parallel to the axes of rotation of the supports.

Finally it is also possible to produce oblique transformations by reason of the fact that the dulled glass plate or photographic plate holder, as also the frame of the original, can be shifted also in a vertical direction, that is to say parallel to the axis of rotation $x\ x^1$ and $y\ y^1$ of the supports of the original and image, and these rotary movements can be measured by division marks. Or by reason of the fact that the dulled glass screen or holder, as also the frame of the original, can be shifted in a horizontal position, that is to say perpendicularly to the axis of rotation $x\ x^1$ and $y\ y^1$ of the supports of the original and image and these shifting movements can be measured by means of division marks; it is possible to alter the scale of the projective alteration of the picture. By this means the apparatus is rendered capable of producing any kind of alteration of image, projective or affinite.

The nature of this constructional form is not altered in any way when, as is usual in large installations, the original is placed in a brightly lighted chamber and the support of the image is in a dark room and the objective is situated in the partition separating the two rooms ; or if the supports of the original and of the image are constructed in the form of complicated portable frameworks for the purpose of producing the necessary movements and for the purpose of producing correct lighting effects. Or if for this purpose or with the object of enabling variable increases to be made within wide limits, the positive actuating mechanism is constructed in a different manner for the purpose of producing a constant clearness of definition of the image.

Fig. 19 shows diagrammatically and Figs. 19^a and 19^b illustrate the practical construction of an apparatus designed only for exactly determined but rather large alterations of image in a relatively small sized picture. It has externally the form of a three-sided prism. In one edge M is mounted an axle on which can turn the objective wall M O in the interior of the apparatus. The image $a^1 b^1 c^1$ which it is desired to alter is inserted in the turn table U and is placed within its axis of collineation parallel to the edge M of the apparatus. When the apparatus has been rectified the desired transformation $a^{11} b^{11} c^{11}$ will at once appear on the dulled glass screen d^1 of the apparatus.

Fig. 20 shows diagrammatically an apparatus for allowing of transformations within much wider limits. In its basic form it is similar to the apparatus shown in Fig. 19, but the two planes M A and M B are capable of coarse and fine adjustment relatively to the objective plane M O through large angles by means of the circular bar T^1. Its common axis of rotation is the so-called axis of distortion M. The light-proof closure is effected by devices such as bellows, etc., which allow of large alterations of dimensions. The mode of operation of this apparatus is the same as that of the preceding apparatus except that it has a greater scope of degrees of distortion.

Fig. 21 shows an apparatus designed again only for exactly determined moderate transformations but for larger sizes of picture than the apparatus shown in Fig. 19. Externally it has the form of a four-sided prism. The planes M A and M B, and the plane M O of the objcective intersect one another in a straight line (M) outside of the apparatus. The plane of the objective can be shifted in the interior of the apparatus along circular bars T^1 which are concentric to M. Its mode of operation is analogous to that of the two apparatus just before described.

Fig. 22 illustrates an apparatus which again allows of effecting alterations of a picture within rather wide limits. In, this basic form it is similar to the apparatus shown in Fig. 21, but the walls A A^1 and B B^1 are connected by

means of bellows, *etc.,* and are therefore rendered movable. The walls A A^1 and B B^1can be shifted with coarse and fine adjustment relatively to the objective wall M O through large angles along circular bars T^1 which are concentric to M. The common axis of rotation M is situated. outside of the apparatus. The mode of operation of this apparatus is also analogous to that of the two preceding forms.

In the apparatus described with reference to Figs. 19, 20, 21, and 22, the original $a^1b^1c^1$ and the objective O, and the projection image $a^{11}b^{11}c^{11}$ are adjustable with fine and coarse adjustment in slides perpendicularly to the axis of rotation M within wide limits, and the amount of this shifting may be measured by division marks, the zero marks of which, lie in the, axis of distortion M of the apparatus.

The original is further mounted in a turn-table U which is capable of rotating with coarse and fine adjustment on an axis perpendicular to the respective plane A or B. Division marks along the slides having their zero points situated in the axis of distortion M, and also divided circles for measuring the angles *e* and *i,* through which the rotating walls can be rotated, allow of an exact adjustment of the distance of the counter axis from the axis of collineation, of the distance of the objective from the axis of collineation, and of the angles *e* and *i,* which the planes of the original and image must make with the plane of the objective.

The type of apparatus to which the above four apparatus belong and which is characterised by the fact that the planes of the objective and image intersect one another at all times actually in a straight line by reason of the construction of the apparatus, allows, like the slide apparatus, of the use of a positive actuating mechanism which secures the permanent fulfillment of also the second condition for obtaining clearness of definition of the image, whereby an apparatus is provided which gives a constant clear definition and thereby considerably simplifies the work.

Two constructional forms of these apparatus will now be described.

The apparatus illustrated in Figs. 23, 23^a and 23^b consists of a hollow three-sided prism M B V, of which the angle *i* at M (the axis of collineation) may be any desired angle. In the wall M V a photographic objective O is capable of being moved to and fro in a straight guide. A dulled glass screen or plate holder or a diapositive frame d^1 may be inserted in the wall MB, The edge M also constitutes the axis for a rotary wall M A which is connected by means of a light-proof bellows to the prism wall M V and is provided along its entire length with a straight guide for a dulled glass screen or a holder or a diapositive frame d^2.

The rotary motion of the wall M A is connected with the to and fro motion of the objective O in the wall M V in a positive manner such that the distance O g_1 from the optical centre (the first nodal point) of the objective to the counter-axis (g_1) in the plane M A (sensitive film), measured parallel to the wall M B, remains constant $(O\ g_1 = \frac{F}{\sin i})$.

The arrangement for the positive movement of the objective O which is dependent on the rotary motion of the wall M A, consists of a right angle triangle *n* which is fixed to the latter and which is similar to the triangle M V B and which slides with its longer cathetus in a straight guide along the wall M V, whilst the pivot $g_1$in the plane M A can slide in a slot guide so that this plane as well as the support d^1(dulled glass screen, holder, or frame) will share all the motions of the objective O.

For the purpose of effecting rectifications, the pivot $g_1$is not arranged direct on the triangle *n* but on the end of a straight edge o^1 which can be shifted along the hypotenuse of the triangle *n* and be fixed by means of a set screw o^2.

As will be readily seen this compulsory connection may also be effected in many other ways. As further the angle *i* may vary and may also be of dif-

ferent size, the general rule for this compulsory connection is that by its means the counter axes, that is to say, the straight lines of intersection of the planes drawn through the optical nodal points ($h_1 h_{11}$) of the objective parallel to the planes A and B, with the latter planes must be obtained in the focal planes of the objective O.

Further the axis of collineation M or the edge M of the prism, instead of being real as in the examples herein described, may be merely imaginary if they cannot be brought within the apparatus by reason of want of space.

For the same reason it is immaterial for the nature of the apparatus whether the supports $d^1 d^2$ (dulled glass screen, holder, or diapositive frame) be fixed in the walls M A and M B, or be made movable in the direction of the axis of collineation M or perpendicularly to the same, or whether they are capable of rotating on axes which are perpendicular to the planes of the original and image.

Apparatus of this kind are suitable not only for affinite distortions, but also for generally projective distortions of all kinds and degrees.

When an original is transformed from the plane M A on to the plane M B, and then after shifting the objective O and the plane M A, it is transformed back on to the latter, then the two pictures in the plane M A will be affinite at right angles to each other when the picture on the plane M B (intermediate image) has remained exactly in the same place in both operations, whilst the dimension in the direction of the axis M remains unaltered.

But the two pictures on the plane M A will be obliquely affinite to each other when the intermediate image on the plane M B has been shifted in the direction of the axis M. Finally they become generaIly projective to each other when the intermediate image has been shifted perpendicuarly to the axis M.

A simple and generally projective, distortion is produced when only one of the two operations is carried out, and the transformation is effected from the plane M A on to the plane M B, or reversely from M B on to M A. But complicated projective distortions are produced when the axes of collineation intersect or cross one another, if the intermediate image was rotated on an axis which is perpendicular to the plane of the intermediate image M B.

It is also of advantage to connect together in a compulsory manner not only the rotary motion of the planes M A and MB, but also the shifting movements. of the supports $d^1 d^2$ (dulled glass screen, plate holder, frame) with the movements of the objective O.

This can be done very easily by causing any desired point or part of the support d^1 to describe a compulsory movement parallel to the plane of the objective (for instance by means of a straight guide $f^2 g^2$, parallel to the objective plane M V at any desired distance from the latter, or by connecting the support d^1 to the pivot o^1.

The above described apparatus is based on the assumption that the angle i which the plane M B makes with the objective plane M V can be kept constant and can be rendered invariable by construction. This of course limits the utility of this apparatus, and it is of advantage to have a construction of the positive actuating mechanism which allows of rendering both planes (original and image) capable of rotation.

This requirement is satisfied in the apparatus illustrated in Figs. 24, 24[a] and 24[b]; having the shape of a book which is provided with the original and image planes capable of rotating on both sides and with a positive actuating mechanism, and it satisfies the requirements.

Its construction is as follows :— The base board on which the apparatus is mounted carries at a distance equal to twice the focal length. (2 F) of the objective employed from the objective plane O M on both sides of it a straight guide $f^2 g^2$ and $h^2 k^2$ with a spindle driving mechanism J and J^1 in which a nut K and K^1 works to and fro. These two nuts K and K^1 are provided each with a rotary guide sleeve L L^1 in which a two armed lever E slides with its

ends and has its pivot e^1 situated vertically under the optical centre of the objective O and which has the effect of causing these two nuts and the objective to remain at all times in a plane. These two nuts K K^1 are further pivoted to slide blocks which slide in straight guides *r s* and *v w* of the planes M A and M B that are pivoted on the common axis of distortion M of the apparatus and which carry with them the supports $d^1 d^2$ which are capable of moving in the picture walls.

The mode of operation of this apparatus resulting from this construction is a very simple one. As each of the spindles J J^1 has a certain amount of friction they can never slide passively, but can only act actively. Consequently the nut K or K^1 which is not operated by its own spindle J or J^1 will therefore always constitute a point of support and a pivot for the entire positive actuating mechanism. When therefore the angle *e* which the plane of the original M A must enclose with the plane of the objective M O has been first adjusted with the help of the actuating mechanism (in which operation it is not at all necessary to trouble about the second spindle drive J^1 in any way) then the adjustment of the angle *e* has been finally effected and is only necessary to adjust the angle *i* which the plane M B of the transformation must enclose with the plane M O of objective with the help of the second spindle drive J^1. In this manner the objective O has automatically moved into its former position. The images are clearly defined and show the desired alteration.

An affinite alteration of image may be considered as being the sum or the difference of two or more projective transformations. Since in many cases the intermediate image has no practical value, and its production entails time and labour, and constitutes a not unappreciable cause of mistakes, it has been proposed by arranging two apparatus in series, *i.e.,* one behind the other, to produce the desired affinite alteration of image in one operation.

The problem of producing directly affinite alterations of image without fixing an intermediate image is always dependent on the condition that the counter axes of the intermediate image which appertain to the two optical systems shall coincide in a straight line. But this may be effected in a great variety of ways, namely,

1. By collimation of two optical systems with each other that is to say by such arrangement of the said systems that their focal planes shall coincide. Any desired plane image can be reproduced by such a combination only in an affinite manner, because the plane of the intermediate image can intersect the common focal plane only in a straight line. Since the ratio of the scales of the two affinite pictures is always determined by the ratio of the distances of the axes of collineation of the two picture systems from the common counter axis, therefore this ratio of the scales is fixed in this case by the ratio between the focal lengths of the two optical systems;

2. By causing the focal plane of one optical system to coincide with the counter plane of the other optical system, and by ensuring that the counter plane of the first system and the focal plane of the other system shall intersect the latter plane which is common to both optical systems in a straight line. In such a case all pictures having their planes parallel to the counter plane of the first system will be reproduced affinitely and the affinitely altered images will lie in planes which are parallel to the counter plane, of the second system. But so long as the distances of the axes of collineation of the two pictures systems from the common counter axes are variable, then the scale of the affinite image will be variable also.

3. The two optical systems may be arranged at any desired angle to each other.

As the counter axes of the intermediate image meet each other in a straight line and must be situated in the focal planes of the two optical systems, only an affinite image can be produced when the straight line of intersection of two focal planes becomes the common counter axis of the intermediate image.

From this follows that the mutual or relative position of the two optical systems to each other determines the position of the two counter planes, and thereby also determines the position in space of these picture planes parallel to the said counter planes if a directly affinite image is to be reproduced at all. When the two optical systems are connected rigidly with each other, then this position in place of the two picture planes is invariable. When the relative position of the two optical systems is altered, either by rotation or by parallel movement, the counter planes will rotate and likewise also the picture planes.

The foregoing considerations form the basis for the hereinafter described constructional forms of apparatus for producing directly affinite alterations of image.

Figs. 25 and 25[a] illustrate an apparatus comprising two hollow mirrors S_1 and S_2 mounted at a fixed angle to each other, one of the two mirrors (S_2) being capable of movement along its vertex plane. By this arrangement the counter axes g_3 of the intermediate image is fixed invariably as the straight line of intersection of the two focal planes F_1 and F_2 in space. Similarly in the case of the counter plane $C_1 g_3$ of the stationary mirror S_1. The counter plane $C_2 g_3$ of the movable mirror S_2 rotates within narrow limits on the counter axis g_3. When it is desired to obtain merely images in the natural size the plane C of the intermediate image must be so drawn through $M_1 g_3 = M_2 g_3$, that is to say, the distances of the two axes of collineation from the counter axis are equal to each other.

By this means the plane A of the original $a^1 b^1$ is also unalterably fixed because it must be drawn through the axes of collineation M_1 parallel to the counter plane $C_1 g_3$, and the plane of the affinitely altered image is determined by the plane which is drawn through the axis of collineation M_2 parallel to the counter plane $C_2 g_3$. This plane is also capable of rotating only within narrow limits like the counter plane $C_2 g_3$.

Fig. 25 illustrates diagrammatically the path of the rays for two positions of the mirror S_2. Fig. 25[a] shows also diagrammatically the constructional arrangement of the apparatus.

The mirror S_1 and the support A A of the original $a^1 b^1$ are mounted rigidly on a baseboard N in the relative position which has been found by Fig. 25.

The mirror S_2 and the axis of rotation X of the supports B B of the transformed image $a^{11} b^{11}$ must on the contrary be mounted on the movable board Q, in such a manner that. the axis of rotation X shall lie in the focal plane F_2 of the mirror S_2.

The support B B is capable of rotation within narrow limits on the axis X. The base board of the mirror S_1 and that of the mirror S_2 are connected in such a manner that the mirror S_2 and the axis X can be shifted parallel to the vertex plane of the mirror S_2 relatively to S_1.

The frame of the original $a^1 b^1$ is freely movable along A A, and the frame of the transformed image $a^{11} b^{11}$ is freely movable along B B. Suitable means such as bellows or the like are provided for the purpose of rendering the whole light-proof.

In use the original is placed in its support A A, so that an image is produced on B B which can be rendered clearly defined by rotating B. B on the axis X. By shifting the mirror S_2 and the axis X with the actuating screw provided for that purpose and by constantly keeping the image clearly defined by rotating B B as required the desired affinite alteration of image will be produced.

Fig. 26 shows the combination of a lens with a mirror and also an example of an arrangement in which one of the focal planes of the objective is caused to coincide with the counter plane of the mirror. Further by the fact that not only the counter axes but also the axes of collineation of the intermediate image are common in both picture systems, that is to say the distance of the axes of collineation from the counter axis is the same in both picture systems,

the. scale is kept constant and the reproduction in the natural size is ensured.

In its external shape the apparatus is similar to the apparatus shown in Fig. 23, At the place where the intermediate picture is fixed by photography in the latter there is in this case a spherical hollow mirror built in the apparatus. The original a^1b^1 is mounted so as to be capable of movement in the wall M V of the box M V B and also capable of rotation in a turn table. The hollow mirror S_1 is mounted on the wall M B in such a manner that M B forms its vertex plane. It reflects the original a^1b^1 reduced, inverted and projectively altered as a virtual image $a^{111}b^{111}$ in the plane M D. This virtual image is received by the objective O which is movable in the wall M V and is projected thereby into the plane M A. The image $a^{11}b^{11}$ produced at that place is a directly affinite alteration of image of the original a^1b^1; the degree of affinity may be altered by shifting the original a^1b^1 and the objective O. The straight line connecting the centre of the mirror with the centre of the objective O determines the direction of the affinity rays. The picture plane M A rotates on the common straight line of intersection (M) of the four planes M V, M B, M D, M A. The plane M A is, as in the apparatus shown in Fig. 23, moved positively in all the movements of the objective O, so that a constantly clearly defined image is likewise produced.

Figs. 27, 27[a] and 27[b] illustrate a combination of two mirrors collimated with each other, that is to say, of two mirrors having such relative positions that their focal planes coincide with each other. The ratio of focal lengths of the two optical systems is 1:1, consequently the affinite image is produced also in this case in natural size.

The two mirrors collimated with each other are mounted in the box that contains the supports of the original a^1b^1 and of the image $a^{11}b^{11}$ which is to be affinitely altered, and which are capable of rotating on parallel axes of rotation x and y situated in the focal plane of the mirror. The original a^1b^1 is illuminated from H. The rays pass through a screen and fall upon the mirror S_1 which reflects them with the production of an intermediate image (that need not further be considered) on to the mirror S_2 which combines them again to form the affinitely altered image $a^{11}b^{11}$.

When the geometrical locus of the original in all possible positions is a circular cylinder, then, as shown in Fig. 27, the geometrical locus of all possible positions and dimensions of the affinitely altered image is an elongated elliptical cylinder.

Constant clearness of definition may be obtained also with this apparatus, by coupling the rotary motions of the two image supports a^1b^1 and $a^{11}b^{11}$ by means of positive driving mechanism in such a manner that their image planes will always intersect in the same straight line a plane supposed to pass through the centres of curvature C_1 and C_2 of the two mirrors S_1 and S_2 parallely to the axes of rotation x and y;

This can be proved with reference to Fig. 27[a]. The plane of the intermediate image (M_1M_2) must always be parallel to the two counter planes C_1x and C_2y.

Since $M_1M_2 = 2\ C_1x = 2\ C_2y$; and the triangle $M_1M_2M_2^1$ is similar to the triangle $C_1x\ M_2^1$: then $C_1M_2^1 = C_1M_2^1$.

Since the triangle $M_1M_2M_1^1$ is similar to the triangle $C_2y\ M_1^1$; therefore $C_2M_1 = C_2M_1^1$.

Since further the triangle $C_2M_1^1m^2$ is similar to the triangle $C_1M_2m^2$; therefore $C_2M_1^1 : C_1M_2^2 = C_2m^2 : C_1m^2$ ($C_2M_1^1 = C_1M_2$) ; $C_1M_2 = C_1C_2 : C_1m^2$.

Likewise the triangle $C_2M_1m^1$ is similar to the triangle $C_1M_2^1m^1$; therefore $C_2M_1 : C_1M_2^1 = C_2m^1 : C_1m^1$.

Further $(C_2M_1 - C_1M_2^1) : C_1M_2^1 = C_1C_2 : C_1m^1$.

Since $C_2M_1^1 - C_1M_2 = C_2M_1 - C_1M_2^1$, therefore $C_1m^1 = C_1m^3\ C^1m$.

This point m is again very suitable for use in construction, as shown in Fig. 27[b], where J is a screw spindle arranged below or above the apparatus in

a plane passing through the centres of curvature C_1 and C_2 parallelly to the axes of rotation x and y. This screw spindle J actuates a nut K^1 to-and-fro in a dove-tail guide. Pivoted to this nut K^1 are two guide links L in which slide arms P that are rigidly connected to the image supports a^1b^1, $a^{11}b^{11}$ in such a manner that their axes lie in the image planes of the image support. The result of this arrangement is to convert every movement of the nut K^1 into rotary motion of the two image supports, whilst the condition for the clearness of definition of the image is constantly kept fulfilled.

Finally, Figs. 28 and 28[a] illustrate apparatus in which the directly affinite alteration of image is produced by a combination of lenses and in which the rear focal plane F_2 of the first optical system is caused to coincide with a counter plane $O_2 g_3$ of the second optical system by rotating the lens O_2, and in which further by the fact that the three planes of the original, the intermediate image, and the affinite image, intersect one another in a straight line, the unalterability of the scale of the image is ensured as well as the production of an image in the natural size.

Fig. 28 illustrates the theory of the instrument with the omission of all concrete parts (lenses, walls, *etc.*). Fig. 28[a] however illustrates diagrammatically the constructional arrangement of the apparatus. The tele-objective O_1 (Fig. 28[a]) with the principal planes H_1 and H_2 (Fig. 28) produces with the obliquely placed original a^1b^1 the projective, intermediate image $a^{111}b^{111}$. This image $a^{111}b^{111}$ is transformed into the virtual image $a^{11}b^{11}$ which is affinite to the original a^1b^1, by means of a lens O_2 capable of rotation on an axis which is drawn through the rear focus of the tele-objective parallel to the axis of rotation x of the original a^1b^1 before it is converted into a virtual image. This virtual image is finally projected by the lens system O_3 as an inverted real image $a^{1111}b^{1111}$ of the same dimensions on to the picture plane B B.

In order that the image may become an affinite image, the following, conditions must be satisfied: —

1. The optical centre of the rotary lens O_2 must coincide with the rear focus of the front system O_1.

2. The focal plane f_2 of the rotary lens O_2 must pass through the counter axis g_3 of the intermediate image which has already been determined by the inclination of the original a^1b^1.

3. The original a^1b^1 must be moved parallel to itself at such a distance $x\ h_1$ from the tele-objective, that the straight line of intersection M_2 of the objective plane of O_2 with the second principal plane of the tele-objective H_2 and the straight line of intersection M_1 of the plane A of the original a^1b^1 with the first principal plane H_1 of the tele-objective shall become conjugate straight lines in an optical sense.

These three conditions may also be stated mathematically by the following equations:—

$$\frac{F}{\tan e} = \frac{f}{\sin Z}\ ;\ \text{and}\ \frac{F}{\tan Z} = \frac{x\,h_1}{\tan e};$$ where F = the

equivalent focal length of the tele-objective : f = the focal length of the rotary lens ; e = the angle enclosed between the plane of the original and the principal planes of the tele-objective: Z = the angle enclosed between the objective plane of the rotary lens and the principal planes of the tele-objective; and $x\ h_1$ = the distance of the axis of rotation x of the original a^1b^1 from the first nodal point h_1 of the tele-objective.

Consequently in the constructional arrangement of the apparatus the following provisions are made for three movements: —(a) the lens O_2 is capable of rotating on an axis y, and its angular rotation can be read off on a divided circle. (b) Similarly the original is capable of rotation on the axis x and its angular rotation can be read off on a divided circle. (c) Finally, the frame of the original a^1b^1 with its axis x is capable of axial movement and the dis-

tances of the axis x from the first nodal point h_1 of the tele-objective O_1 can be read off on a longitudinal scale.

All the remaining parts are essentially rigidly connected together and are made movable or adjustable solely for purposes of rectification.

The optical systems O_1 and O_3 are tele-objectives for the purpose of moving the lenses themselves as near as possible to each other and thus increasing the field of vision by utilising the fact that the principal planes of such systems are situated far outside of the lenses.

-Dated this 16th day of January 1904

For the Applicant
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COMPLETE SPECIFICATION.

Improved Method and Apparatus for the Systematic Alteration or Distortion of Plane Pictures and Images by means of Lenses and Mirrors for Photography and for other purposes.

I, THEODOR SCHEIMPFLUG, of 39 Sternwartestrasse, Vienna XVIII, Austria, Captain; do hereby declare the nature of this invention and in what manner the same is, to be performed to be particularly described and ascertained in and by the following statement:—

It is well known in the practice of photography that in reproductions, slight alterations of the original picture can be readily effected without affecting the clearness of definition of the photographic picture by slightly inclining the original picture and the receiving plate relatively to the objective. Reproducing apparatus are also known which allow of utilizing this circumstance. But this alteration of the original picture has hitherto been done solely in an empirical manner and it is a very tedious and troublesome operation, which can be carried out only within very narrow limits and with a comparatively small degree of accuracy. The best apparatus for producing alterations of pictures which I know, is the "amplificateur rectifier" of Mr. Carpentier of Paris; but even that apparatus enables only the rectification of obliquely taken photographs to be effected, and it cannot be used for other purposes and it has therefore only a very limited scope of use. For this reason it has often been preferred to attempt to obtain the alteration of the original picture without reference to the clearness of definition of the photographic picture, and to obtain the requisite clearness of definition of the photographic picture by the use of very small screens and strong light.

But on the other hand it is a matter both of scientific as well as of commercial interest to be able to carry out all possible alterations of the original picture in an exact and systematic manner, and the present invention has for its object to provide an improved method based on exact and mathematical principles, of solving this problem, and also improved apparatus for carrying out the said improved method.

In the drawings filed with my Provisional Specification, Figs. 1 and 2 are respectively a section and a perspective view illustrating the meaning of the terms "projective" and "perspective" "axis of collineation" and "axis of projection".

Figs. 3 and 4 illustrate in perspective view and in section the meaning of the terms "counter plane" and "counter axis", and the consequent geometrical relations of the two collinear pictures to each other.

Fig. 5 illustrates the conditions of oblique reproduction, so far as they are known.

Fig. 6 illustrates the mutual perspective relations of several projective images, the planes of which intersect one another in the same straight line.

Figs. 7, 8 and 9 illustrate the condition of affinity; that is to say:–

Fig. 7 illustrates the case in which all the images remain upright;

Fig. 8 ilustrates the case in which there is first an inversion of the image and then the image is re-erected, and, Fig. 9 illustrates the case in which first an erect image is produced which is then inverted, and *vice versa.*

Figs. 10 and 11 illustrate the optical conditions for the production of images in planes inclined to the optical axis with the use of spherical lenses, and the connection between the geometrical relations and the optical relations of the picture and its image thus produced, said connection constituting the basis of the improved method.

Fig. 10 illustrates the use of a symmetrical objective, in the case where the two principal planes coincide with the plane of the objective. Fig. 11 illustrates the case of an unsymmetrical objective, and more particularly of a tele-objective, where the distance between the two principal planes can no longer be neglected.

Figs. 12 and 13 illustrate the laws of the production of images when spherical mirrors are employed.

Figs. 14 and 15 illustrate the procedure which forms the basis of the improved method for the production of affinite pictures by optical means, that is to say, Fig. 14 illustrates the case where the positions of the original and of the photographic picture are convertible; and Fig. 15 illustrates a case where the original can be inserted on one side only.

Fig. 16 illustrates a way of carrying out the improved method for the purpose of direct production of affinite alterations of the original picture.

The remaining figures illustrate various constructional forms of apparatus adapted to carry out the improved method under various given conditions.

Figs. 17, 17^a, 17^b and 17^c are respectively a side elevation, a plan and two detailed views of what is known as a slide apparatus for projective alterations of image. Figures 18 and 18^a serve to illustrate the principle of the positive actuating mechanism employed in the said apparatus. Fig. 19 is a diagram, and Fig. 19^a is a sectional view of a practical construction of apparatus designed for the production of alterations of image only within certain limits, but with great accuracy. Fig. 19^b illustrates as an appurtenant detail a turn-table which is also employed in all the other apparatus. Figs. 20, 21 and 22 illustrate diagrammatically modifications of the apparatus shown in Fig. 19.

Figs. 23, 23^a and 23^b are respectively a plan, a side elevation, and a section of a constructional form of apparatus which secures the permanent fulfilling of the second condition of clearness of definition of image and which has resulted from the original form shown in Fig. 19. Figs. 24, 24^a, and 24^b, are respectively a plan, a horizontal section and a vertical section of a second constructional form of apparatus fulfilling this condition and arranged in the form of a book. Figs. 25 and 25^a are diagrammatic representations of an apparatus which is adapted to produce affinite alterations of image without the production of an intermediate image, and in which only mirrors are employed. Fig. 26 shows likewise diagrammatically an apparatus which produces in a direct manner affinite alterations of image and in which a mirror is combined with an objective. Figs. 27, 27^a and 27^b illustrate in like manner an apparatus for directly affinite alterations of image in which only mirrors are employed. Figs. 28 and 28^a illustrate an apparatus for directly affinite alterations of image in which only lenses are employed.

In accordance with the newer geometry or geometry of position, two plane figures *a, b, c* . . . . and a^1, b^1, c^1, (Figs. 1 and 2) which have been produced by the intersection of the same bundle of rays by two planes A and B, are

described as being "projective" or "collinear" to each other, and every relative position of those planes in which all the straight lines connecting two points $a\ a^1$, $b\ b^1$, $c\ c^1$, of the respective planes, meet in one point o, is called a perspective position. The point *c* from which these connecting straight lines (lines of sight) diverge, is called the centre of projection or of collineation, and the straight line of intersection M M of the two planes A and B which is common to the two figures under consideration, is called the axis of collineation. Since the axis of collineation is common to the two figures, every straight line of the original will intersect its image in a point in the axis of collineation and *vice versa*.

Again, the planes A^1 and B^1 (Figs. 3 and 4) drawn through the centre of projection *o,* parallel to the two planes A and B, are called counter planes, and they intersect the planes of the pictures in the images of the infinitely distant straight lines of the planes B and A. These lines g^1 and g^2 are called counter-axes and are the geometrical loci of the points of intersection of all the mutually intersecting straight lines which appear as parallel straight lines in the plane of the other picture. That is to say the counter axis g^1 situated in the plane A is the geometrical locus of the points of intersection of all the straight lines which appear as parallel straight lines in the plane B, and vice versa.

An affinite alteration of an original, that is to say, an alteration such that the image is lengthened or shortened in a determined direction, but remains completely, unaltered in another direction, would be possible in a direct manner, such as by the incidence of shadow, only when the centre of projection is situated at an infinite distance. That is to say, plane pictures on the same scale which have an affinite relationship to one another, may first be conceived as figures of intersection of two planes with one and the same cylinder of rays. But the geometry of position teaches that yet another conception is possible. If according to Fig. 6 a plane picture in the plane M N be projected from the point o^1 on to the plane M Q and also from the point o^{11} on to the plane M R, and the three planes M N, M Q, and M R intersect one another in one and the same straight line M M, which is the common axis of collineation of the three pictures, then the two images projected on the planes M Q and M R will be situated in perspective to each other, and their common centre of collineation o^{111} will be situated in the straight line $o^1 o^{11}$. This new centre of collineation o^{111} may however be also situated at an infinite distance. When this is the case the pictures on M Q and M R are in affinite relationship to each other (Fig. 7). Affinity is merely a particular case of projection. However, notwithstanding this fact, for the sake of brevity, the term "projective" will hereinafter be employed, as opposed to "affinite", in the sense that the geometrical relations which are being considered herein, will always be termed "projective," when the centre of projection o^{111} is situated at a finite distance, and will be termed "affinite" when the centre of projection o^{111} is situated at an infinite distance.

"The following axioms which combine optics and the new geometry, constitute the theoretical basis of my improved method and of my improved apparatus for carrying out the same.

1. Photographic images are perspective images in a strictly mathematics sense, and the principal (nodal) points of the objective that is employed, serve as the centre of projection of their corresponding images; that is a well known fundamental fact of "photogmmetry".

2. Every plane image is reproduced as a plane image by systems of spherical lenses or mirrors arranged in any combination, within the limits of accuracy to which the general lens equation or mirror equation is applicable. This is also a well known fact.

3. In oblique reproduction it is well known that the plane of the original and the first principal plane of the system of lenses, as also the plane of the

projection image and the second principal plane of the system of lenses intersect each other in optically conjugate straight lines (Fig. 5).

If the two nodal points $h_1 h_{11}$ combine to form the optical centre O, whereby the two principal planes are caused to coincide with the objective plane M O, the two image planes and the objective plane intersect in the same straight line M which serves as the geometrical axis of collineatlon of the two images.

The general lens equation assumes the form: —

$$\tan e = \frac{F}{b_0 - F} \tan. i.$$

where e and i are the angles enclosed by the image planes and the two principal planes or the objective plane. F is the focus of the objective namely,

+ F for converging lenses.

– F for diverging lenses.

b_0 is the distance of the point of intersection of the optical axis of the objective with the image plane A of the original from the first nodal point h_1 of the objective.

In this connection it should be noted that in most of the systems of lenses in practical use which are symmetrical as a rule, the two nodal points and therefore also the principal plane are situated so close to each other that they can be considered practically as coincident. Then it is allowable to speak of the "optical centre" and of the "plane of the objective" (Fig. 10). Only the tele-systems, that is to say, the combination of positive and negative lenses form an important exception to this rule, by reason of their considerable symmetry, and in these systems, the nodal points or principal planes lie for outside of the system of lenses, namely on the side of the positive lens, whilst the two nodal points or principal planes are shifted far apart at the same time (Fig. 11).

Now the foregoing lens equation has a geometrical significance which is very important for the improved method. Namely, if the counter axes $g_1 g_2$ of the two perspective images a, b, c, a^1, b^1, c^1, are determined by placing through the two nodal points h_1 h_{11} of the objective, planes parallel to the two image planes A and B (Fig. 10), and causing the former planes to intersect the image planes, then these counter axes are always situated in the focal planes f_1 F_{11} of the lens system. The counter axes have thus not only a geometrical significance, but also an optical significance.

Further, the following axiom from the geometry of position, is of importance for the production of affinite alterations of image, namely that two optically perspective systems of images which are required to complement each other, must have the intermediate image and the appurtenant counter axis in common (Figs. 7, 8, 9).

In the case of spherical mirrors (Figs. 12 and 13), the two image planes A and B and the vertex plane S of the mirror intersect one another in the same straight line which fills the function of the geometric axis of collineation of the two images.

The general equation for a mirror (vertex equation) is as follows: —

$$\frac{+ l}{- F} = \frac{l}{R.\tan e} + \frac{l}{R.\tan i}; \text{ if } R = M S, \text{ and } F = \frac{r}{2} =$$

half the radius of curvature.

+ F refers to concave mirrors. — F refers to convex mirrors.

This mirror equation has the analogous geometrical signification as hereinbefore set forth: —

Namely, if the counter axes $g_1 g_2$ of the perspective system of images be determined by drawing planes through the centre of curvature C of the mirror parallel to the two image planes A and B, and causing them to intersect the said two image planes, then these counter axes will always be situated in the focal plane F of the mirror.

It is upon the combination of these laws (part of which were known before, but the greater part of which are new) that the method of altering images as hereinafter described, as well as the apparatus relating thereto are based.

In carrying out the method of producing projective alterations of images, the procedure is as follows: —

After being determined with the help of the laws and principles hereinbefore set forth by mathematical or by graphic calculation, the necessary data, for adjusting the original, the objective, and the image, then when the tele-systems are employed as the objective, the first plane A containing the original, and the second plane B containing the dulled glass screen intended to receive the projected image, together with the corresponding (first or second) principal plane H_1H_{11} of the objective, are caused to intersect one another in such a manner that the two straight lines of intersection thereby produced will become optically conjugate straight lines in accordance with the laws of optics. In this manner the first condition for the clearness of definition of the image is fulfilled. Then one plane, say for instance the plane containing the dulled glass screen is rotated relatively to the other plane which in this case contains the original and which is kept. stationary, until it is parallel to the counter plane which appertains to the aforesaid stationary plane and which is determined by the latter and by the focal plane (appertaining to it) of the objective. In this manner the second condition for the clearness of definition of the image is fulfilled, namely that the counter axes shall lie in the focal planes. Then an image. of the original having the desired degree of distortion and with perfect clearness of definition, will appear on the dulled glass screen.

When symmetrical systems of lenses are used as an objective, in which the two principal planes thereof become the objective plane, the procedure is exactly the same, because the two planes that contain the original and the dulled glass screen are caused to intersect the objective plane in the same straight line and then one plane is rotated relatively to the other stationary plane until it is parallel to the counter plane of the said stationary plane.

When spherical mirrors are used, the foregoing also holds good as in symmetrical objectives, except that in the case of spherical mirrors the vertex plane of the mirror replaces the objective plane.

When affinite alterations of an image are to be produced, the procedure may be as follows:—the devices mentioned hereinbefore are arranged two or even more in number, one behind the other, so that the result of the first device, namely the intermediate image, is either first fixed by photography and then afterwards altered, or it may be immediately further altered by the second device without being so fixed. The same applies also to the result of this second device and its alteration in using the third device; and so on.

Figs. 14, and 15 serve to explain more fully the first method of affinite alteration of an image.

As shown in Fig. 14, the image plane Z of the intermediate image $a^{11}b^{11}c^{11}$, or the dulled glass screen of the first picture is kept stationary at any desired angle (i) relatively to the optical axis of the objective and the carrier of the intermediate image is shifted parallel to the objective plane towards the axis of collineation M or away from the same, according as it is desired to produce a shortening or a lengthening of the proportions of the image. Then the plane of the original is adjusted in such a manner as to cause the two planes and the objective plane to intersect one another in a straight line, and to bring the counter axis g^1 of the original within the focal plane. As soon as the image is clearly visible on the dulled glass screen, the first picture is taken and fixed by photography; then the negative thus obtained is placed in the apparatus in the place from which it was taken. If this negative were now projected back into the plane of the original without any further adjustments, the result will be an exact reproduction of the original. But if the negative be shifted without altering the angle of inclination i to the objective plane (mirror plane)

and without altering the distance from the latter, or if the objective (the mirror) be shifted in its plane without altering the position of the intermediate image (negative), then on projecting back affinite alterations of pictures can be produced in the most varied stages of alteration, after having fulfilled the conditions required for the production of a clearly defined image, by rotating the plane in which the original was originally situated, and on to which the affinitely altered image is being projected back.

This method presumes that the picture to be reproduced and the dulled glass screen, may exchange places relatively to the objective. If this is not the case and if the negative must be fastened to the carrier of the original in order to carry out the second alteration, then the method undergoes a modification. The intermediate image (negative) $a^{11}b^{11}c^{11}$ is then, as shown in Fig. 15, arranged by the side of the original in such a manner that the counter axis g^1 of the original which was determined at the first "taking" as the straight line of the intersection of the plane Z with the focal plane (F_{11}), is now caused by the new adjustment to come within the focal plane (F_1). The shifting of the parts parallely to the objective plane and the exact adjustment are effected in the same manner as in the preceding case.

But in the present case every alteration in the angle of inclination (i) of the intermediate image relatively to the objective plane, causes an alteration in the scale of the affinite alteration.

As above stated, the method for the production of complicated projective and affinite alterations of an image may also be carried out without taking a photograph of the intermediate image, by immediate further alteration of the same, and the necessary adjustments may be effected by correspondingly inclining or rotating two or more optical systems.

Direct affinite alterations of images may likewise be effected, for example, without taking a photograph, of an intermediate image, by immediate further alteration of the latter, by collimating two optical systems (two lenses, one lens and one mirror, or two mirrors) with each other, that is to say, causing their focal planes to coincide with each other.

Two such systems collimated with each other, are unable to reproduce oblique pictures otherwise than as affinite pictures, because the plane of the intermediate image can intersect the common focal plane of the two optical systems in a single straight line only, that is to say, in the common counter axis, thus by itself fulfilling the condition for affinity.

Fig. 16 illustrates the carrying out of this method with the use of two hollow mirrors S and S^1. The original $a\ b$ is set within the field of vision of the two mirrors at any desired angle to the focal plane. The mirror S by reflecting the light rays from the original on to the mirror S^1, produces an imaginary or virtual intermediate image situated outside of the surface of the figure and which is transformed by the mirror S^1 into the affinitely altered image a^1b^1.

The practical carrying out of the method of projective alteration of an image is considerably facilitated by the use of apparatus which automatically give a constant clearness of definition of image, because then instead of the preliminary calculations and constructions, only the following simple rules need to be observed: —

1. Rotation of the original serves to place the straight lines which are to remain parallel in the original as well as in the distorted image, parallel to the axis of collineation, that is to say, it serves to place the axis of collineation of the original parallel to the axis of distortion of the apparatus, and then by shifting the image to bring it into coincidence with the axis of distortion.

2. Parallel straight limes in the original appear in the transformation as straight lines intersecting one another, and *vice versa.* The geometrical loci of the points of intersection are the counter axes.

3. The degree of the divergence or of the convergence of mutually intersect-

ing images of parallel straight lines of the original and *vice versa,* is solely a function of the distance of the counter axes of the respective images from the axes of collineation. If this distance is smaller, then the divergence will be greater, when the distance is greater the divergence will be less.

4. The distance between the images of straight lines which are parallel to the axis of collineation, is in the transformation, smaller or larger than in the original, that is to say, the transformation appears to be shortened or lengthened in comparison with the original when it is shifted nearer to or further away from the axis of collineation than the original.

5. A rectangle in the original appears in the image as a trapeze when it is symmetrical to the optical axis of the objective; and it appears as a trapezoid when it is unsymmetrical and the angles of the trapezoid are the more dissimilar, the greater the unsymmetry of the position of the trapezoid.

6. The distance of the counter axis of the transformation from the axis of collineation which distance is the determining factor for the divergence or the convergence of the projective images of parallel straight lines in the original, is best regulated continuously by rotating the plane containing the transformed image, but it may also be altered intermittently by changing the objective, because it is also dependent on the focal length of the objective.

I shall now proceed to describe some constructional forms of apparatus adapted to carry out the improved method hereinbefore set forth.

The first step in the provision of practical apparatus was the adaptation of photographic reproducing apparatus in such a manner that the plane of the original and the plane of its image can be inclined to the optical axis of the objective. But in apparatus of this kind the clearness of definition of the image is the sole criterion of the proper adjustment. Consequently such apparatus are incomplete, inconvenient, and not capable of accurate work. An essential and industrially new improvement on these known constructions consists in the provision of sighting devices in the planes of the original and its image which allow of controlling the exact intersection of the two planes with the objective plane in the same straight line, that is to say, of satisfying the first condition of clearness of definition. Another way of attaining the same object but which entails a special modification of ordinary photographic reproducing apparatus when employed for these purposes and which constitutes a great constructional improvement consists in combining the holders or supports of the objective and of the original and its image constructionally in such a manner that they will always intersect one another in a straight line. Further by taking care that the second condition for clearness of definition of the image is satisfied automatically, then types of apparatus are evolved which render all preparatory calculation unnecessary in the case of projective alterations of image because the clearness of definition of the image is already constantly provided for, and as the projective alterations of the image in such apparatus take place according to very simple and clear laws, and as they are the only matter which the operator has to consider they may readily be carried out in a purely empiric manner. The apparatus which satisfy automatically the two conditions of clearness of definition of the image are therefore industrially the most valuable part of the present invention. But for the most industrial purposes projective alterations of image are generally of less importance than affinite alterations of image. In the latter case apparatus are important which enable affinite alterations of image to be effected without any intermediate process. These have been produced constructively by the combination or the arrangement in series of two apparatus for projective alterations in which the intermediate image (which together with its counter axis is common to both projective systems) remains merely an aerial or imaginary image.

The apparatus shown in Fig. 17 in side elevation and in Fig. 17[a] in plan consists of a photographic reproducing apparatus which is provided with all the devices that are necessary to carry out projective alterations of image in

a systematic manner. For this purpose the frames *r s* and *v w* which are designed to receive the original and the plate holder for the image are arranged to rotate on vertical axes *x* x^1 and *y* y^1 that are mounted in the supports *f g* and *h k*. These supports are capable of being shifted rectilinearly and perpendicularly to the axis of the objective.

The shifting of the supports *f g* and *h k* parallel to the plane of the objective and in opposite directions at the same time, takes place along the straight guides T T and is effected by the lever E which is pivoted on the pin e^1 and is actuated by means of a screwed spindle I and a nut K. The rotary motions of the plane of the original and the plane of the image relatively to each other and to the plane of the objective are effected by means of a positive actuating mechanism in such a manner that both the conditions of clearness of definition of the image are satisfied, and therefore when the apparatus has been once rectified, it will produce permanently clearly defined images.

The said positive actuating mechanism consists of a screwed-spindle J which is arranged in the objective plane and drives a nut K^1 carrying two pivoting guides L in which can slide arms P rigidly connected to sleeves S mounted loose on the parallel rotary axes *x* x^1 and *y* y^1 of the supports *r s* and *v w*.

The rotary motion of the sleeves S is transmitted by means of suitable mechanism (such as for instance bevel wheel gearing G) in the same amount but in the opposite direction of rotation to the rotary frames *r s* and *v w*.

The mode of operation of this positive actuating mechanism is illustrated by Fig. 18.

The first condition to be satisfied in order to produce clearness of definition of the image is the following:—The plane of the original, the plane of the image, and the plane of the objective must always intersect one another in a straight line M.

As in the great majority of cases only slight inclinations of the plane of the original and the plane of the image are required, this straight line M is situated as a rule far outside the apparatus and it is therefore difficult to utilize it for the purposes of construction. Consequently, it is necessary to substitute for this straight line M another but more easily followed mark *m*.

If, as shown in Fig. 18, the distances of the axes of rotation *x* x^1 and *y* y^1 from the plane of the objective M O are made equal to each other and to twice the focal length of the objective employed, and if

$$\text{the angle O x } m^1 = \text{angle O Mx} = e$$
$$\text{angle Oym}^2 = \text{angle Omy} = i$$

then the triangle O x M is similar to the triangle O m x. The triangle O y M is similar to the triangle O m y, because all their angles are equal; therefore

$$\text{O M. O } m^1 = \text{Ox}^2$$
$$\text{O M. O m}^2 = \text{Oy}^2$$

But as it was assumed that O x = O y, then O m^1= O m^2= O m; that is to say m^1 and m^2 meet at the point *m*. Now this point *m* is very convenient for use for the purposes of construction, and its place is taken by the nut K^1. The rotary arms P take the place of the lines *mx* and *my,* and the supports *r s* and v w take the place of the lines M x and M y. The bevel wheel gearing or other mechanism employed makes the angles of rotation *e* and *i* of the arms P and of the supports *r s* and *v w* equal to each other and opposite in direction. Further the arms P by reason of their pivoting guides L meet always at the point *m* of the plane of the objective. Consequently the picture planes of the supports *r s* and *v w* will be caused to intersect each other always in the straight line M in the plane of the objective.

But since O x = O y; then the counter planes O g_1 and O g_2 divide the

triangle x M y into the two similar and equal triangles x g_1 O and O g_2 y and the parallelogram O g_1 M g_2; and

$$x\ g_1 = g_1 M, \text{ and}$$
$$y\ g_2 = g_2 M.$$

The straight lines g_1 and g_2 are situated at half the distance of the straight lines $x\ x^1$ and $y\ y^1$ from the plane of the objective, and they lie therefore in the focal planes.

In this manner the two conditions for producing clearness of definition of the images are strictly satisfied.

Fig. 18[a] illustrates a modification of the positive actuating mechanism just described.

In this case the distance of the axes of rotation $x\ x^1$ and $y\ y^1$ from the plane of the objective are variable, but they must also in this modification be capable of being moved to and fro in straight guides parallelly to the plane of the objective.

If as shown in Fig. 18[a];

$$\text{angle } Ox\ m^1 = \text{angle } O\ My = i$$
$$\text{angle } Oym_2 = \text{angle } OMx = e$$

then triangle O x M is similar to triangle O y m^2 and triangle O y M is similar to triangle O x m^1, therefore

$$\frac{O\ M}{O\ x} \cdot \frac{O\ x_1}{O\ m} = \frac{\sin a \,.\, \sin b}{\sin e \,.\, \sin i} = \frac{O\ M.}{O\ y} \frac{Oy_2}{Om}$$

Consequently O m^1 = O m^2 = O m, that is to say m^1 and m^2 meet at the point *m*.

From this it follows that: If the point *m* be determined in the manner shown in Fig. 18[a], namely by drawing the angle *e* at *y* and the angle *i* at *x* on the opposite side of M, then the distances of the axes of rotation $x\ x^1$ and $y\ y^1$ need no longer be equal to each other and to twice the focal length of the objective employed.

But the positive actuating mechanism requires to be arranged in a somewhat different manner:–A nut K^1 with pivoting guides moves along the plane of the objective. The arms P sliding in the guides of the nut K^1, rotate the sleeves S which are mounted loose on the axes $x\ x^1$ and $y\ y^1$. But the rotary movements of these loose sleeves S are now no longer transmitted to the supports for the original and image mounted on the same axis, but are transmitted by means of suitable mechanism (cord gearing, parallelo guides &c.) to the oppositely situated supports for the original and image in equal angles but in opposite directions.

With this arrangement the clearness of definition is not absolutely ensured as in the arrangement before described, and it is maintained only when it has been once obtained. Namely, in Fig. 18[a], the counter axes g_1 and g_2 will move strictly parallel to the plane of the objective, so long as *x* and *y* are likewise moved parallel to the plane of the objective. Consequently, when these counter axes have been set in the focal planes, by having once obtained the clearness of definition of the image, they will remain in those planes, and the clearness of definition of the image will likewise be maintained.

For the purpose of also satisfying the highest requirements, and of facilitating an exact rectification of the apparatus, the following arrangement is further provided.

Two slit-aperture lamps *p,* which are respectively keyed fast on the vertical axles of rotation $x\ x^1$ and $y\ y^1$ of the supports *r s* and *v w,* throw beams of light constantly in the direction of the planes of the original and of the image. When all the parts are in the middle position, those beams of light must

coincide with the sights provided on the supports *f g* and *h k,* and when the supports *r s* and *v w* are inclined sufficiently, they must meet in a straight line M on the screen *u* situated in the plane of the objective.

In cases where the requisite inclinations of the planes of the original and image are so great that the positive actuating mechanism no longer suffices, the positive actuating mechanism may be thrown out of engagement, and then the beams of light will serve also as sighting and adjusting means for causing the two planes to intersect the plane of the objective always in the straight line M.

Certain devices are also provided for the purpose of rendering the apparatus capable of solving even the more difficult problems which can be controlled by mathematical calculation. Such devices are:—

Division-marks which indicate the respective distances of the vertical axes of rotation $x\ x^1$ and $y\ y^1$ from the optical axis of the objective; divided circles which allow of reading off the angles *e* and *i* through which the supports *r s* and *v w* are rotated; a turntable U constructed in the support *r s* so as to allow, when inserting a picture into the apparatus, of setting the axis of collineation or the counter axis of the picture parallel to the axes of rotation of the supports.

Finally it is also possible to produce oblique transformations by reason of the fact that the dulled glass plate or photographic plate holder, as also the frame of the original, can be shifted also in a vertical direction, that is to say parallel to the axis of rotation $x\ x^1$ and $y\ y^1$ of the supports of the original image, and these rotary movements can be measured by division marks. Or by reason of the fact that the dulled glass screen or holder, as also the frame of the original, can be shifted in a horizontal position, that is to say perpendicularly to the axis of rotation $x\ x^1$ and $y\ y^1$ of the supports of the original and image and these shifting movements can be measured by means of division marks; it is possible to alter the scale of the projective alteration of the picture. By this means the apparatus is rendered capable of producing any kind of alteration of image, projective or affinite.

The nature of this constructional form is not altered in any way when, as is usual in large installations, the original is placed in a brightly lighted chamber and the support of the image is in a dark room and the objective is situated in the partition separating the two rooms; or if the supports of the original and of the image are constructed in the form of complicated portable frameworks for the purpose of producing the necessary movements and for the purpose of producing lighting effects. Or if for this purpose or with the object of enabling variable increases to be made within wide limits, the positive actuating mechanism is constructed in a different manner for the purpose of producing a constant clearness of definition of the image.

Fig. 19 shows diagrammatically and Figs. 19^a and 19^b illustrate the practical construction of an apparatns designed only for exactly determined but rather large alterations of image in a relatively small sized picture. It has externally the form of a three-sided prism. In one edge M is mounted an axle on which can turn the objective wall M O in the interior of the of the apparatus. The image $a^1b^1c^1$ which it is desired to alter is inserted in the turn table U and is placed within its axis of collineation parallel to the edge M of the apparatus. When the apparatus has been rectified the desired transformation $a^{11}b^{11}c^{11}$ will at once appear on the dulled glass screen d^1 of the apparatus.

Fig. 20 shows diagrammatically an apparatus for allowing of transformations within much wider limits. In its basic form it is similar to the apparatus shown in Fig. 19, but the two planes M A and M B are capable of coarse and fine adjustment relatively to the objective plane M O through large angles by means of the circular bar T^1. Its common axis of rotation is the so-called axis of distortion M. The light-proof closure is effected by devices such as

bellows, *etc.,* which allow of large alterations of dimensions. The mode of operation of this apparatus is the same as that of the preceding apparatus except that it has a greater scope of degrees of distortion.

Fig. 21 shows an apparatus designed again only for exactly determined moderate transformations but for larger sizes of picture than the apparatus shown in Fig. 19. Externally it has the form of a four-sided prism. The planes M A and M B, and the plane M O of the objective intersect one another in a straight line (M) outside of the apparatus. The plane of the objective can be shifted in the interior of the apparatus along circular bars T^1 which are concentric to M. Its mode of operation is analogous to that of the two apparatus just before described.

Fig. 22 illustrates an apparatus which again allows of effecting alterations of a picture within rather wide limits. In its basic form it is similar to the apparatus shown in Figs. 21, but the walls A A^1 and B B^1 are connected by means of bellows, *etc.,* and are therefore rendered movable. The walls A A^1 and B B^1 can be shifted with coarse and fine adjustment relatively to the objective wall M O through large angles along circular bars T^1 which are concentric to M. The common axis of rotation M is situated outside of the apparatus. The mode of operation of this apparatus is also analogous to that of the-two preceding forms.

In the apparatus described with reference to Figs. 19, 20, 21, and 22, the original $a^1 b^1 c^1$ and the objective O, and the projection image $a^{11} b^{11} c^{11}$ are adjustable with fine and coarse adjustment in slides perpendicularly to the axis of rotation M within wide limits, and the amount of this shifting may be measured by division marks, the zero marks of which lie in the axis of distortion M of the apparatus.

The original is further mounted in a turn-table U which is capable of rotating with coarse and fine adjustment on an axis perpendicular to the respective plane A or B. Division marks along the slides having their zero points situated in the axis of distortion M, and also divided circles for measuring the angles *e* and *i,* through which the rotating walls can be rotated, allow of an exact adjustment of the distance of counter axis from the axis of collineation, of the distance of the objective from the axis of collineation, and of the angles *e* and *i,* which the planes of the original and image must make with the plane of the objective.

The type of apparatus to which the above four apparatus belong and which is characterized by the fact that the planes of the objective and image intersect one another at all times actually in a straight line by reason of the construction of the apparatus, allows, like the slide apparatus, of the use of a positive actuating mechanism which secures the permanent fulfilment of also the second condition for obtaining clearness of definition of the image, whereby an apparatus is provided which gives a constant clear definition and thereby considerably simplifies the work.

Two constructional forms of these apparatus will now be described.

The apparatus illustrated in Figs. 23, 23^a and 23^b consists of a hollow three-sided prism M B V, of which the angle *i* at M (the axis of collineation) may be any desired angle. In the wall M V a photographic objective O is capable of being moved to and fro in a straight guide. A dulled glass screen or plate holder or a diapositive frame d^1 may be inserted in the wall M B. The edge M also constitutes the axis for a rotary wall M A which is connected by means of a light-proof bellows to the prism wall M V and is provided along its entire length with a straight guide for a dulled glass screen or a holder or a diapositive frame d^2.

The rotary motion of the wall M A is connected with the to and fro motion of the objective O in the wall M V in a positive manner such that the distance O g_1 from the optical centre (the first nodal point) of the objective to the counter-axis (g_1) in the plane M A (sensitive film), measured parallel to the wall M B, remains constant $\left(O\ g_1 = \frac{F}{\sin i}\right)$.

The arrangement for the positive movement of the objective O which is dependent on the rotary motion of the wall M A, consists of a right angle triangle n which is fixed to the latter and which is similar to the triangle M V B and which slides with its longer cathetus in a straight guide along the wall M V, whilst the pivot g^1 in the plane M A can slide in a slot guide so that this plane as well as the support d^1 (dulled glass screen, holder, or frame) will share all the motions of the objective O.

For the purpose of effecting rectifications, the pivot g_1 is not arranged direct on the triangle n but on the end of a straight edge o^1 which can be shifted along the hypotenuse of the triangle n and be fixed by means of a set screw o^2.

As will be readily seen this compulsory connection may also be effected in many other ways. As further the angle i may vary and may also be of different size, the general rule for this compulsory connection is that by its means the counter axes, that is to say the straight lines of intersection of the planes drawn through the optical nodal points ($h_1 h_{11}$) of the objective parallel to the planes A and B, with the latter planes must be obtained in the focal planes of the objective O.

Further the axis of collineation M or the edge M of the prism, instead of being real as in the examples herein described, may be merely imaginary if they cannot be brought within the apparatus by reason of want of space.

For the same reason it is immaterial for the nature of the apparatus whether the supports $d^1 d^2$ (dulled glass screen, holder, or diapositive frame) be fixed in the walls M A and M B, or be made movable in the direction of the axis of collineation M or perpendicularly to the same, or whether they are capable of rotating on axes which are perpendicular to the planes of the original and image.

Apparatus of this kind are suitable not only for affinite distortions, but also for generally projective distortions of all kinds and degrees.

When an original is transformed from the plane M A on to the plane M B, and then after shifting the objective O and the plane M A, it is transformed back on to the latter, then the two pictures in the plane M A will be affinite at right angles to each other when the picture on the plane M B (intermediate image) has remained exactly in the same plane in both operations, whilst the dimension in the direction of the axis M remains unaltered.

But the two pictures on the plame M A will be obliquely affinite to each other when the intermediate image on the plane M B has been shifted in the direction of the axis M. Finally they become generally projective to each other when the intermediate image has been shifted perpendicularly to the axis M.

A simple and generally projective distortion is produced when only one of the two operations is carried out, and the transformation is effected from the plane M A on to the plane M B, or reversely from M B on to M A. But complicated projective distortions are produced when the axes of collineation intersect or cross one another, if the intermediate image was rotated on an axis which is perpendicular to the plane of the intermediate image M B.

It. is also of advantage to connect together in a compulsory manner not only the rotary motion of the planes M A and M B, but also the shifting movements of the supports $d^1 d^2$ (dulled glass screen, plate holder, frame) with the movements of the objective O.

This can be done very easily by causing any desired point or part of the support d^1 to describe a compulsory movement parallel to the plane of the objective (for instance by means of a straight guide $f^2 g^2$, parallel to the objective plane M V at any desired distance from the latter, or by connecting the support d^1 to the pivot o^1.

The above described apparatus is based on the assumption that the angle i which the plane M B makes with the objective plane M V can be kept constant and can be rendered invariable by construction. This of course limits

the utility of this apparatus, and it is of advantage to have a construction of the positive actuating mechanism which allows of rendering both planes (original and image) capable of rotation.

This requirement is satisfied in the apparatus illustrated in Figs. 24, 24[a] and 24[b]; having the shape of a book which is provided with the original and image planes capable of rotating on both sides and with a positive actuating mechanism, and it satisfies the requirements.

Its construction is as follows: The base board on which the apparatus is mounted carries at a distance equal to twice the focal length (2 F) of the objective employed from the objective plane O M on both sides of it a straight guide f^2g^2 and h^2k^2 with a spindle driving mechanism J and J^1 in which a nut K and K^1 works to and fro. These two nuts K and K^1 are provided each with a rotary guide sleeve L L^1 in which a two armed lever E slides with its ends and has its pivot e^1 situated vertically under the optical centre of the objective O and which has the effect of causing these two nuts and the objective to remain at all times in a plane. These two nuts K K^1 are further pivoted to slide blocks which slide in straight guides *r s* and *v w* of the planes M A and M B that are pivoted on the common axis of distortion M of the apparatus and which carry with them the supports d^1d^2 which are capable of moving in the picture walls.

The mode of operation of this apparatus resulting from this construction is a very simple one. As each of the spindles J J^1 has a certain amount of friction they can never slide passively, but can only act actively. Consequently the nut K or K^1 which is not operated by its own spindle J or J^1 will therefore always constitute a point of support and a pivot for the entire positive actuating mechanism. When therefore the angle *e* which the plane of the original M A must enclose with the plane of the objective M O has been first adjusted with the help of the actuating mechanism (in which operation it is not at all necessary to trouble about the second spindle drive J^1 in any, way) then the adjustment of the angle *e* has been finally effected and is only necessary to adjust the angle *i* which the plane M B of the transformation must enclose with the plane M O of objective with the help of the second spindle drive J^1. In this manner the objective O has automatically moved into its former position. The images are clearly defined and show the desired alteration.

An affinite alteration of image may be considered as being the sum or the difference of two or more projective transformations. Since in many cases the intermediate image has no practical value, and its production entails time and labour, and constitutes a not unappreciable cause of mistakes, it has been proposed by arranging two apparatus in series, *i.e.,* one behind the other, to produce the desired affinite alteration of image in one operation.

The problem of producing directly affinite alterations of image without fixing an intermediate image is always dependent on the condition that the counter axes of the intermediate image which appertain to the two optical systems shall coincide in a straight line. But this may be effected in a great variety of ways, namely:

1. By collimation of two optical systems with each other that is to say by such arrangement of the said systems that their focal planes shall coincide. Any desired plane image can be reproduced by such a combination only in an affinite manner, because the plane of the intermediate image can intersect the common focal plane only in a straight line. Since the ratio of the scales of the two affinite pictures is always determined by the ratio of the distances of the axes of collineation of the two picture systems from the common counter axis, therefore this ratio of the scales is fixed in this case by the ratio between the focal lengths of the two optical systems;

2. By causing the focal plane of one optical system to coincide with the counter plane of the other optical system, and by ensuring that the counter plane of the first system and the focal plane of the other system shall intersect

the latter plane which is common to both optical systems in a straight line. In such a case all pictures having their planes parallel to the counter plane of the first system will be reproduced affinitely and the affinitely altered images will lie in planes which are parallel to the counter plane of the second system. But so long as the distances of the axes of collimation of the two pictures systems from the common counter axes are variable, then the scale of the affinite image will be variable also.

3. The two optical systems may be arranged at any desired angle to each other.

As the counter axes of the intermediate image meet each other in a straight line and must be situated in the focal planes of the two optical systems, only an affinite image can be produced when the straight line of intersection of two focal planes becomes the common counter axis of the intermediate image. From this follows that the mutual or relative position of the two optical systems to each other determines the position of the two counter planes, and thereby also determines the position in space of these picture planes parallel to the said counter planes if a directly affinite image is to be reproduced at all. When the two optical systems are connected rigidly with each other, then this position in place of the two picture planes is invariable. When the relative position of the two optical systems is altered, either by rotation or by parallel movement, the counter planes will rotate and likewise also the picture planes.

The foregoing considerations form the basis for the hereinafter described constructional forms of apparatus for producing directly affinite alterations of image.

Figs. 25 and 25[a] illustrate an apparatus comprising two hollow structures S_1 and S_2 mounted at a fixed angle to each other, one of the two mirrors (S_2) being capable of movement along its vertex plane. By this arrangement the counter axes g_3 of the intermediate image is fixed invariably as the straight line of intersection of the two focal planes F_1 and F_2 in space. Similarly in the case of the counter plane $G_1 g_3$ of the stationary mirror S_1. The counter plane $C_2 g_3$ of the movable mirror S_2 rotates within narrow limits on the counter axis g_3. When it is desired to obtain merely images in the natural size the plane C of the intermediate image must be so drawn through $M_1 g_3 M_2 g_3$, that is to say, the distances of the two axes of collineation from the counter axis are equal to each other.

By this means the plane A of the original $a^1 b^1$ is also unalterably fixed because it must be drawn through the axes of collineation M_1 parallel to the counter plane $C_1 g_3$, and the plane of the affinitely altered image is determined by the plane which is drawn through the axis of collineation M_2 parallel to the counter plane $C_2 g_3$. This plane is also capable of rotating only within narrow limits like the counter plane $C_2 g_3$.

Fig. 25 illustrates diagrammatically the path of the rays for two positions of the mirror S_2. Fig. 25[a] shows also diagrammatically the constructional arrangement of the apparatus.

The mirror S_1 and the support A A of the original $a^1 b^1$ are mounted rigidly on a baseboard N in the relative position which has been found by Fig. 25.

The mirror S_2 and the axis of rotation X of the supports B B of the transformed image $a^{11} b^{11}$ must on the contrary be mounted on the movable board Q, in such a manner that the axis of rotation X shall lie in the focal plane F_2 of the mirror S_2.

The support B B is capable of rotation within narrow limits on the axis X. The base board of the mirror S_1 and that of the mirror S_2 are connected in such a manner that the mirror S_2 and the axis X can be shifted parallel to the vertex plane of the mirror S_2 relatively to S_1.

The frame of the original $a^1 b^1$ is freely movable along A A, and the frame of the transformed image $a^{11} b^{11}$ is freely movable along B B. Suitable means

such as bellows or the like are provided for the purpose of rendering the whole light-proof.

In use the original is placed in its support A A, so that an image is produced on B B which can be rendered clearly defined by rotating B B on the axis X. By shifting the mirror S_2 and the axis X with the actuating screw provided for that purpose and by constantly keeping the image clearly defined by rotating B B as required the desired affinite alteration of image will be produced.

Fig. 26 shows the combination of a lens with a mirror and also an example of an arrangement in which one of the focal planes of the objective is caused to coincide with the counter plane of the mirror. Further by the fact that not only the counter axes but also the axes of collineation of the intermediate image are common in both picture systems, that is to say the distance of the axes of collineation from the counter axis is the same in both picture systems, the scale is kept constant and the reproduction in the natural size is ensured.

In its external shape the apparatus is similar to the apparatus shown in Fig. 23. At the place where the intermediate picture is fixed by photography in the latter there is in this case a spherical hollow mirror built in the apparatus. The original a^1b^1 is mounted so as to be capable of movement in the wall M V of the box M V B and also capable of rotation in a turn table. The hollow mirror S_1 is mounted on the wall M B in such a manner that M B forms its vertex plane. It reflects the original a^1b^1 reduced, inverted and projectively altered as a virtual image $a^{111}b^{111}$ in the plane M D. This virtual image is received by the objective O which is movable in the wall M V and is projected thereby into the plane M A. The image $a^{11}b^{11}$ produced at that place is a directly affinite alteration of image of the original a^1b^1; the degree of affinity may be altered by shifting the original a^1b^1 and the objective O. The straight line connecting the centre of the mirror with the centre of the objective O determines the direction of the affinity rays. The picture plane M A rotates on the common straight line of intersection (M) of the four planes M V, M B, M D, M A. The plane M A is, as in the apparatus shown in Fig. 23, moved positively in all the movements of the objective O, so that a constantly clearly defined image is likewise produced.

Figs. 27, 27^a and 27^b illustrate a combination of two mirrors collimated with each other, that is to say, of two mirrors having such relative positions that their focal planes coincide with each other. The ratio of focal lengths of the two optical systems is 1:1, consequently the affinite image is produced also in this case in natural size.

The two mirrors collimated with each other are mounted in the box that contains the supports of the original a^1b^1 and of the image $a^{11}b^{11}$ which is to be affinitely altered, and which are capable of rotating on parallel axes of rotation x and y situated in the focal plane of the mirror. The original a^1b^1 is illuminated from H. The rays pass through a screen and fall upon the mirror S_1 which reflects them with the production of an intermediate image (that need not further be considered) on to the mirror S_2 which combines them again to form the affinitely altered image $a^{11}b^{11}$.

When the geometrical locus of the original in all possible positions is a circular cylinder, then, as shown in Fig. 27, the geometrical locus of all possible positions and dimensions of the affinitely altered image is an elongated elliptical cylinder.

Constant clearness of definition may be obtained also with this apparatus, by coupling the rotary motions of the two image supports a^1b^1 and a^{11} and b^{11} by means of positive driving mechanism in such a manner that their image planes will always intersect in the same straight line a plane supposed to pass through the the centres of curvature C_1 and C_2 of the two mirrors S_1 and S_2 parallely to the axes of rotation x and y;

This can be proved with reference to Fig. 27[a]. The plane of the intermediate image (M_1M_2) must always be parallel to the two counter planes C_1x and C_2y.

Since $M_1M_2 = 2\ C_1x = 2\ C_2y$; and the triangle $M_1M_2M_2$ is similar to the triangle $C_1x\ M_2^1$; then $C_1M_2 = C_1M_2^1$.

Since the triangle $M_1M_2M_1^1$ is similar to the triangle $C_2y\ M_1^1$; therefore $C_2M_1 = C_2M_1^1$.

Since further the triangle $C_2M_1^1m^2$ is similar to the triangle $C_1M_2m^2$; therefore $C_2M_1^1 : C_1M^2 = C_2m^2 : C_1m^2$ ($C_2M_1^1 = C_1M_2$) : $C_1M_2 = C_1C_2 : C_1m^2$.

Likewise the triangle $C_2M_1m^1$ is similar to the triangle $C_1M_2^1m^1$; therefore $C_2M_1 : C_1M_2^1 = C_2m^1 : C_1m^1$.

Further $(C_2M_1 - C_1C_1M_2^1) : C_1M_2^1 = C_1C_2 : C_1m^1$.

Since $C_2M_1^1 - C_1M_2 = C_2M_1 - C_1M_2^1$, therefore $C_1m^1 = C_1m^2 = C_1m$.

This point m is again very suitable for use in construction, as shown in Fig. 27[b], where J is a screw spindle arranged below or above the apparatus in a plane passing through the centres of curvature C_1 and C_2 parallelly to the axes of rotation x and y. This screw spindle J actuates a nut K^1 to-and-fro in a dove-tail guide. Pivoted to this nut K^1 are two guide links L in which slide arms P that are rigidly connected to the image supports a^1b^1, $a^{11}b^{11}$ in such a manner that their axes lie in the image planes of the image support. The result of this arrangement is to convert every movement of the nut K^1 into rotary motion of the two image supports, whilst the condition for the clearness of definition of the image is constantly kept fulfilled.

Finally, Figs. 28 and 28[a] illustrate apparatus in which the directly affinite alteration of image is produced by a combination of lenses and in which the rear focal plane F_2 of the first optical system is caused to coincide with a counter plane O_2g_3 of the second optical system by rotating the lense O_2, and in which further by the fact that the three planes of the original, the intermediate image, and the affinite image, intersect one another in a straight line, the unalterability of the scale of the image is ensured as well as the production of an image in the natural size.

Fig. 28 illustrates the theory of the instrument with the omission of all concrete parts (lenses, walls, *etc.*). Fig. 28[a] however illustrates diagrammatically the constructional arrangement of the apparatus. The tele-objective O_1 (Fig. 28[a]) with the principal planes H_1 and H_2 (Fig. 28) produces with the obliquely placed original a^1b^1 the projective intermediate image $a^{111}b^{111}$. This image $a^{111}b^{111}$ is transformed into the virtual image $a^{11}b^{11}$ which is affinite to the original a^1b^1, by means of a lens O_2 capable of rotation on an axis which is drawn through the rear focus of the tele-objective parallel to the axis of rotation x of the original a^1b^1 before it is converted into a virtual image. This virtual image is finally projected by the lens system O_3 as an inverted real image $a^{1111}b^{1111}$ of the same dimensions on to the picture plane B B.

In order that the image may become an affinite image, the following conditions must be satisfied:—

1. The optical centre of the rotary lens O_2 must coincide with the rear focus of the front system O_1.

2. The focal plane f_2, of the rotary lens O_2 must pass through the counter axis g_3 of the intermediate image which has already been determined by the inclination of the original a^1b^1.

3. The original a^1b^1 must be moved parallel to itself at such a distance X h_1 from the tele-objective, that the straight line of intersection M_2 of the objective plane of O_2 with the second principal plane of the tele-objective H_2 and the

straight line of intersection M_1 of the plane A of the original a^1b^1 with the first principal plane H_1 of the tele-objective shall become conjugate straight lines in an optical sense.

These three conditions may also be stated mathematically by the following equations:—

$\frac{F}{\tan e} = \frac{f}{\sin Z}$; and $\frac{F}{\tan Z} = \frac{x\ h_1}{\tan e}$; where F = the equivalent focal length of the tele-objective; f = the focal length of the rotary lens; e = the angle enclosed between the plane of the original and the principal planes of the tele-objective; Z = the angle enclosed between the objective plane of the rotary lens and the principal planes of the tele-objective; and $x\ h_1$ = the distance of the axis of rotation x of the original a^1b^1 from the first nodal point h_1 of the tele-objective.

Consequently in the constructional arrangement of the apparatus the following provisions are made for three movements :—(a) the lens O_2 is capable of rotating on an axis y, and its angular rotation can be read off on a divided circle. (b) Similarly the original is capable of rotation on the axis x and its angular rotation can be read off on a divided circle. (c) Finally, the frame of the original a^1b^1 with its axis x is capable of axial movement and the distances of the axis x from the first nodal point h_1 of the tele-objective O_1 can be read off on a longitudinal scale.

All the remaining parts are essentially rigidly connected together and are made movable or adjustable solely for purposes of rectification.

The optical systems O_1 and O_3 are tele-objectives for the purpose of moving the lenses themselves as near as possible to each other and thus increasing the field of vision by utilising the fact that the principal planes of such systems are situated far outside of the lenses.

Having now particularly described and ascertained the nature of this invention and in what manner the same is to be performed I declare that What I claim is:—

1. The hereinbefore described improved method for the production of projective alterations of images by optical or photographic means in which the first image plane or plane containing the original and the first principal plane of the objective on the one hand, and the second image plane or plane containing the image, and the second principal plane of the objective on the other hand are arranged to intersect one another respectively in such a manner that the resulting two straight lines of intersection become optically conjugate straight lines according to the laws of optics, said improved method consisting in rotating the plane containing the original or the image relatively to the other plane containing the image or the original and which is kept stationary, until it is parallel to the counter plane which appertains to the said stationary plane and which is determined by the latter and by the focal plane of the objective, whereby the two geometrical counter axes of the system of images are moved into the focal planes of the objective, as set forth.

2. The improved method as specified in Claim 1, modified for use in those cases in which the two principal planes of the objective may be considered as coinciding with each other and thus constituting the objective plane, and the two planes containing the original and the image are caused to intersect the objective plane in the same straight line, said modification consisting in rotating the plane containing the original or the image relatively to the other plane containing the image or the original and which is kept stationary until it is parallel to the counter-plane which appertains to the said stationary plane, and which is determined by the latter and by the focal plane of the objective, whereby the two geometrical counter-axes of the projective system of images are moved into the two focal plnes of the objective, as set forth.

3. The hereinbefore described improved method for producing projective alterations of an image by optical or photographic means with the use of spherica1 mirrors, in which the two planes containing the original and the image are caused to intersect the vertex plane of the mirror in the same straight line, said improved method consisting in rotating the plane containing the original or the image relatively to the other plane containing the image or the original and which is kept stationary, until it is parallel to the counter plane which appertains to the said stationary plane and which is determined by the latter and by the focal plane of the mirror, whereby the two geometrical counter axes of the projective system of images are moved into the focal plane of the mirror, as set forth.

4. The hereinbefore described improved method for the production of complex projective, and affinite alterations of images by optical or photographic means, which consists in carrying out the connected operations referred to in Claim 1, twice or more times in succession, and, in the case of a combination of only two projective systems of images for the purpose of affinite alteration of image, of causing the counter axes (pertaining to the two systems of images) of the intermediate image to coincide with each other, as set forth.

5. The improved method specified in Claiml 4, with the modification that two or more of the connected operations referred to in Claim 1, are executed at the same time in such a manner that the result of the first operation is immediately altered by the second operation, without being fixed by photography, and, in the case of a combination of of only two projective systems of images for the purpose of affinite alteration of image, of causing the counter axes (pertaining to the two systems of images) of the intermediate image to coincide with each other, as set forth.

6. The improved method specified in Claim 5, with a modification for the direct production of affinite alterations of image, said modification consisting in regulating the operations which are to be executed at the same time, by the oblique setting or by the rotation or shifting relatively to each. other of two optical systems and of the image planes relatively to each other in such a manner that the plane of the intermediate image always appears to pass through the straight line of intesection of two focal planes of the two optical systems, as set forth.

7. The improved method specified in Claim 5, with a modification for the direct production of affinite alterations of image, said modification consisting in regulating the operations which are to be carried out at the same time, by the oblique setting or by the rotation or shifting relatively to each other of two optical systems and of the image planes, in such a manner that one focal plane of one optical system is caused to coincide with a counter plane of the other optical system, whilst the plane of the intermediate image is common to both, as set forth.

8. The improved method specified in Claim 5, with a modification for the direct production of affinite alterations of image, said modification consisting in regulating the operations which are to be carried out at the same time, by causing the focal planes of two optical systems (lenses or mirrors) to coincide with each other adjusting the original to any desired angle to the common focal plane and reproducing the original by the double system obtained by the collimation of the two optical systems, as set forth.

9. Photographic reproducing apparatus and optical instruments for the systematic transformation of plane pictures by the use of spherical lenses or mirrors in which the plane of the original, the projected image, and the plane of the objective, or the vertex plane of the mirror, can intersect one another in a straight line or can be made to intersect each other in such a straight line, and the angles which are enclosed by the three planes with each other namely the plane of the original and the plane of the image and the objective plane or vertex plane of the mirror, can be altered, the arrange-

ment being such that the origiual, the projected image and the objective or the mirror or only one of them can be shifted in their planes; whilst finally at least one of the two pictures (the original) can be placed with its axis of collineation parallel to the axis of distortion of the apparatus by rotating on a turn-table, as set forth.

10. Photographic reproducing apparatus or optical instruments of the kind specified in Claim 9, comprising objectives such that the two principal planes cannot be considered to coincide with the objective plane, in which the plane of the original and the first principal plane of the objective, and the plane of the projected image and the second principal plane of the objective are arranged to intersect one another in parallel straight lines which correspond to each other as object and image according to the laws of optics, or are caused to intersect one another in such planes, and the angles enclosed by the four planes, namely the plane of the original with the first named principal plane, and the plane of the projected image with the second principal plane may be altered, the arrangement being such that the original, the projected image and the objective, or mirror or only one of them, may be shifted in their planes, whilst finally at least one of the two pictures (the original) or both may by rotating on a turn table be placed with its axis of collineation parallel to the axes of distortion of the apparatus, as set forth.

11. In reproducing apparatus with rotary picture walls, the arrangement of any direct or indirect sighting devices for the purpose of causing the planes of the original and image to intersect the objective plane exactly in the same straight line, as set forth.

12. Constructional form of the apparatus specified in Claim 9 as slide apparatus with transverse support for the supports of the original and image capable of rotation on parallel axes, in which the lateral movements and the rotary movements of the supports of the original and image relatively to the objective and to its optical axis are coupled together positively in such a manner that when once the two conditions for clearness of definition of image have been satisfied they will remain automatically satisfied, as set forth.

13. In apparatus according to Claim 12, positive actuating mechanism for producing and maintaining constant clearness of definition of image, comprising a loose sleeve rotated by the to-and-fro movement of a pivoting guide in the objective plane by means of two arms sliding therein on each of the two parallel axes of rotation of the supports of the original and image, the rotary motion of said sleeve being transmitted in equal amounts but in opposite directions to the support mounted on the opposite axis, whilst the two axes of rotation and the optical centre of the objective remain always in one plane, as set forth.

14. In apparatus according to Claim 12, positive actuating mechanism for producing and maintaining constant clearness of definition of image, comprising a loose sleeve rotated by the to-and-fro movement of a pivoting guide in the objective plane by means of two arms sliding therein on each of the two parallel axes of rotation of the supports of the original and image, the rotary motion of said sleeve being transmitted in equal amounts but in opposite directions to the support mounted on the same axis, whilst the two axes of rotation and the optical centre of the objective remain in one plane, aud furthermore the axes of rotation of the supports of the original and image are moved parallel to the objective plane at a distance equal to twice the focal length of the objective, as set forth.

15. Photographic reproducing apparatus and optical instruments for the systematic transformation of plane pictures by the use of spherical lenses or mirrors, in which the planes of the original and image, and the plane of the objective or the vertex plane of the mirror are capable of being rotated on a common axis and therefore intersect one another always in the same straight line, as set forth.

16. A constructional form of the apparatus specified in Claim 15, in which a positive connection is established between the movements of the objective or of the mirror, and the rotary movements of the planes of the original and image, and also if necessary, between the movements of the supports of the original and image along their planes, whereby the geometrical counter-axes are maintained permanently in the focal planes of the objective and of the mirror, and thus constant clearness of definition of image is obtained, as set forth.

17. In apparatus according to Claim 16, positive actuating mechanism for producing and maintaining constant clearness of definition of image, in which the angle enclosed by the plane of the original (or of the image) and the plane of the objective (or mirror) is constant, whilst the plane of the image (or of the original) is capable of rotating on the straight line of intersection of the first mentioned plane with the plane of the objective (or mirror), and is actuated by means of a pivoting guide which is compelled to slide always in the focal plane of the objective (or mirror) and also remains in the counter-plane of the stationary original (or image) the axis of which thus coincides with the counter-axis of the rotatable image (or original), as set forth.

18. The constructional form of apparatus specified in Claim 15, comprising walls capable of rotation towards both sides and devices for producing positive movements for obtaining constant clearness of definition of image with a positive actuating mechanism consisting of the combination of two spindles or straight guides parallel to the plane of the objective or vertex plane of the mirror, fitted with nuts which produce compulsory rotation of the planes of the original and image and also movements of the supports of the original and image in their own planes, with a straight guide for guiding the objective or mirror in its own plane, and with a lever pivoted in a straight line drawn through the optical centre of the objective or centre of curvature of the mirror, parallel to the axis of distortion of the apparatus, said lever keeping the two nuts on the spindles and the objective or centre of curvature of the mirror always in the same plane, as set forth.

19. In apparatus according to Claim 16, positive actuating mechanism for producing and maintaining constant clearness of definition of image, in which the two planes of the original and image which are capable of rotation on a common axis lying in the plane of the objective, are actuated by means of pivoting guides which are themselves moved parallel to the plane of the objective and also maintained together with the optical centre of the objective or the centre of curvature of the mirror in a plane parallel to the axis of rotation of the plane of the original or image, as set forth.

20. Apparatus for producing directly affinite transformations of image, consisting substantially of combinations of two distorting apparatus with lenses or mirrors of the kind specified in the preceding claims, without a common image plane, comprising a pair of systems of lenses or mirrors which are collimated with each other or which are set obliquely or so as to be capable of rotation relatively to each, other, thereby satisfying the condition for producing affinity, *i.e.,* the common imaginary image and its counter axis, as set forth.

21. Apparatus as specified in Claim 20 for the direct production of affinite alterations of image; in which the two optical systems (lenses or mirrors) enclose any desired fixed or variable angle with each other, and the movements of the two image planes are regulated in such a manner that the plane of the intermediate image always appears to pass through the straight line of intersection of two focal planes of the two optical systems as set forth.

22. Apparatus as specified in Claim 20, for the direct production of affinite alterations of image, in which a focal plane of one optical system (lens or mirror) is caused to coincide with a counter-plane of the other optical system (lens or mirror), whilst the plane of the intermediate image is, common to both systems, as set forth.

23. Apparatus as specified in Claim 20 for the direct production of affinite alterations of image, in which the two optical sytems (lenses or mirrors) are

collimated with each other, *i.e.,* have their focal planes brought into coincidence with each other, as set forth.

24. Apparatus as specified in Claim 21 for the direct production of affinite alterations of image, comprising two concave mirrors mounted so as to enclose a determined angle between them only one of which is capable of motion in the direction of its vertex plane, and also one image support is fixed in a determined position relatively to the non-movable mirror, whilst the other image support which shares the movements of the movable mirror in the same sense and to the same extent, is capable of rotating on an axis situated in the focal plane of the movable mirror, as set forth.

25. Apparatus as specified in Claim 22, for the direct production of affinite alterations of image, in which a concave mirror and an objective co-operate with each other, and one image plane coinciding with the objective plane encloses a constant angle with the plane of the concave mirror, whilst the other image plane is capable of rotation on the straight line of intersection of the first image plane (constituting at the same time the objective plane) with the plane of the mirror, and is actuated by a guiding pivot-joint which is always compelled to slide in the focal plane of the objective, while remaining also in the counter plane of the vertical intermediate image whose axis is thus coincident with the counter axis of the rotary image, as set forth.

26. Apparatus as specified in Claim 22 for the direct production of affinite alterations of image, in which only lenses are combined in such manner that two tele-objectives having their negative lenses facing one another are combined with a converging lens rotatable on the rear (inner) focus of one tele-objective, so as to form an optical system which is utilized as the objective of a reproducing apparatus, having one fixed image wall, whilst the other image wall that is situated on the side of the tele-objective whose inner focus serves as the axis of rotation of the converging lens, is capable of rotation on an axis that is parallel to the axis of rotation of the converging lens, and is also capable of movement in the direction of the optical axis of the tele-objective, as set forth.

27. Apparatus as specified in Claim 23 for the direct production of affinite alterations of image, comprising two concave mirrors for affinite alteration of image which are collimated relatively to each other but are shifted relatively to each other along their common focal plane, whilst the image supports rotate on parallel axes of rotation situated in the focal plane of the mirrors, as set forth.

28. In apparatus as specified in Claim 27, positive actuating mechanism for producing and maintaining constant clearness of definition of image, in which the two image supports capable of rotation on parallel axes of rotation situated in the common focal plane of the two-mirrors, are actuated by two co-axial guiding pivot-joints which are moved parallelly to the straight line connecting the centres of curvature of the two hollow mirrors, as set forth.

29. Photographic reproducing apparatus and optical instruments for the systematic transformation or distortion of plane pictures or images by the use of spherical lenses or mirrors, as shown in the illustrative drawings,

Dated this 20th day of February 1904.

For the Applicant

F. WISE HOWORTH, F.C.S,
46 Lincoln's Inn Fields, London, W.C.
Chartered Patent Agent.

Fig.1.

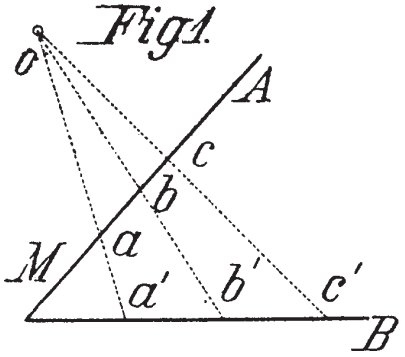


Fig.2.

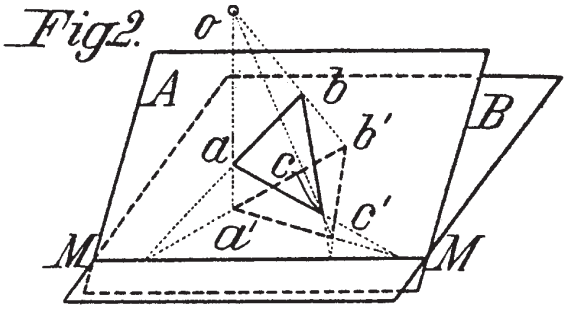
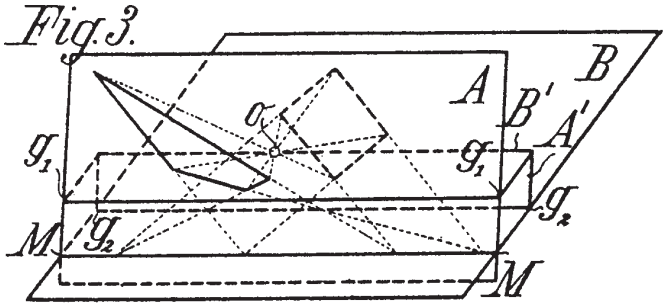


Fig.3.



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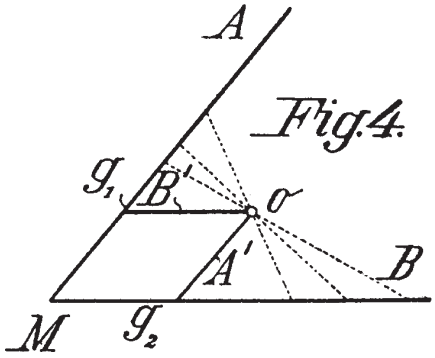

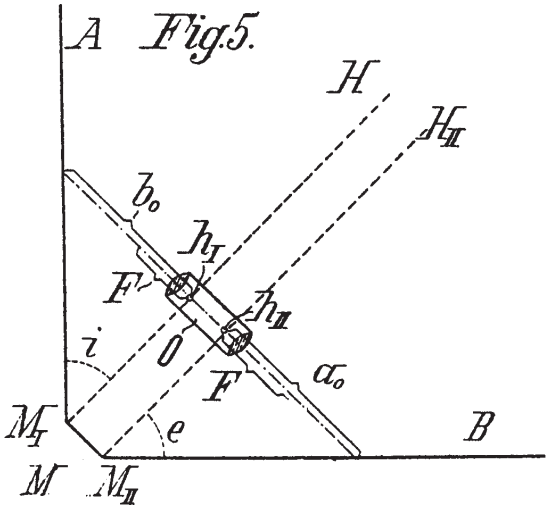

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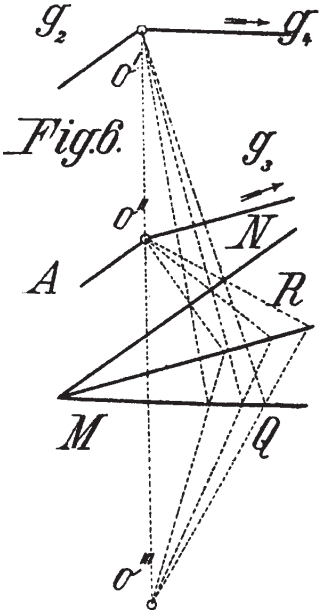

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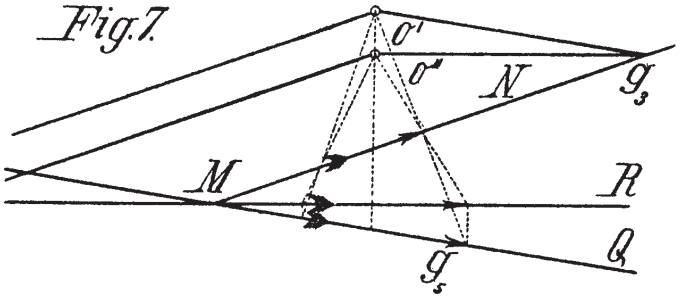
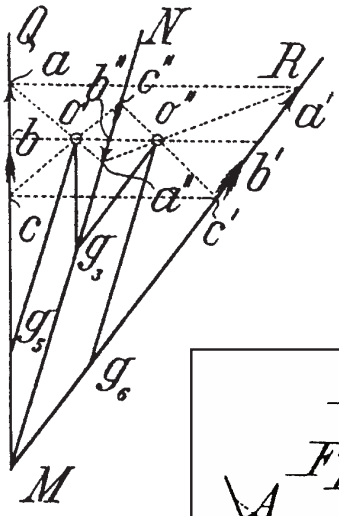


Fig.7.

Fig.8.



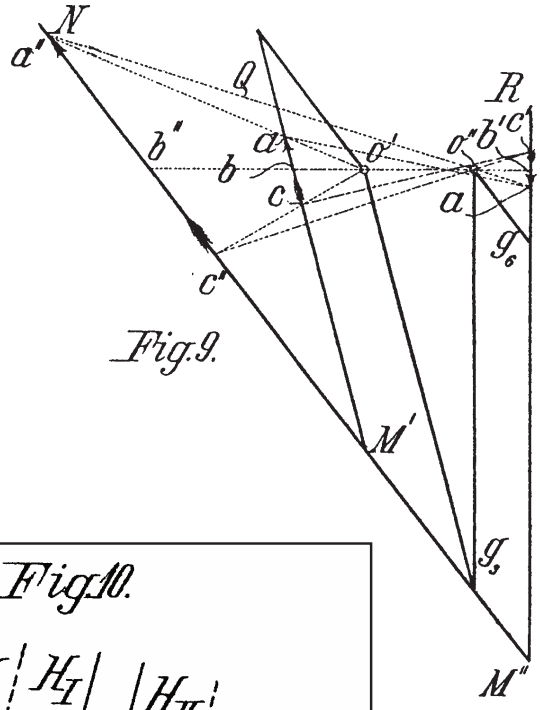
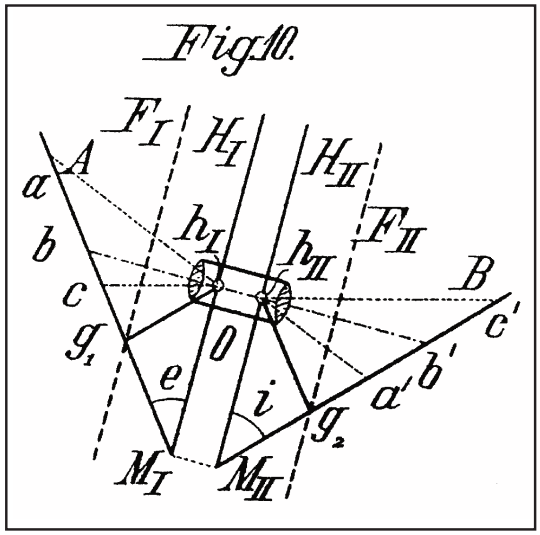


Fig.9.

Fig.10.



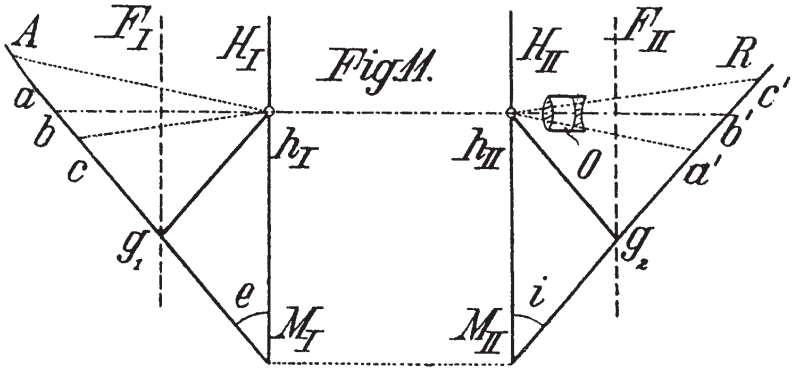
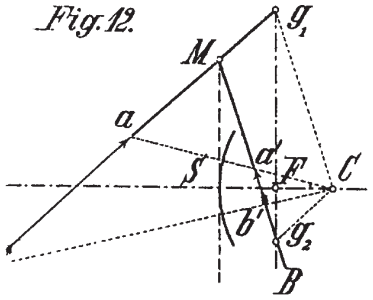


Fig.11.

Fig. 12.



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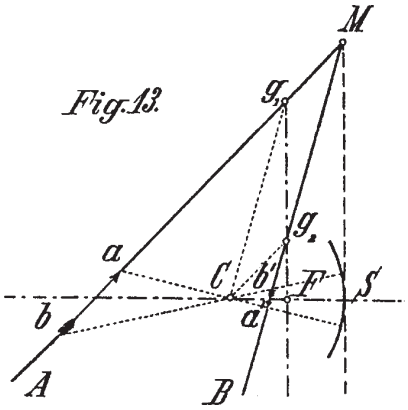

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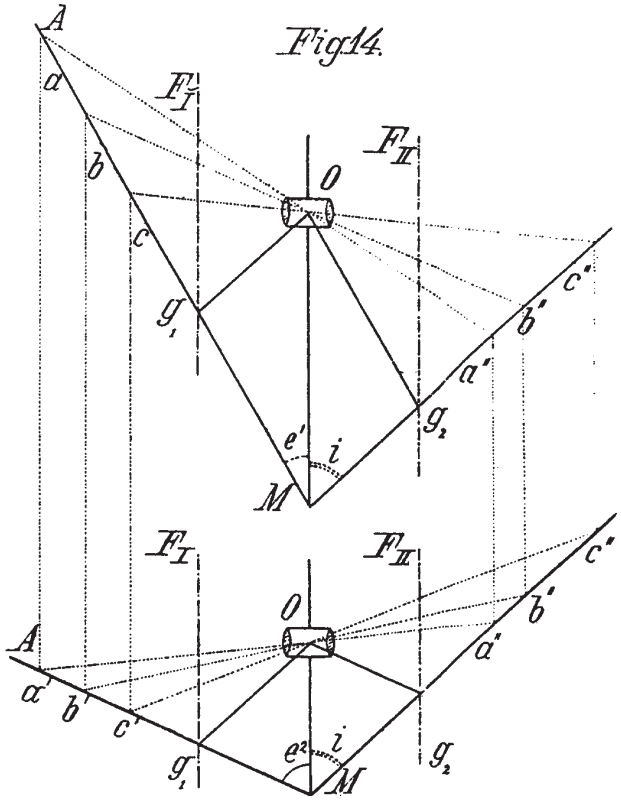

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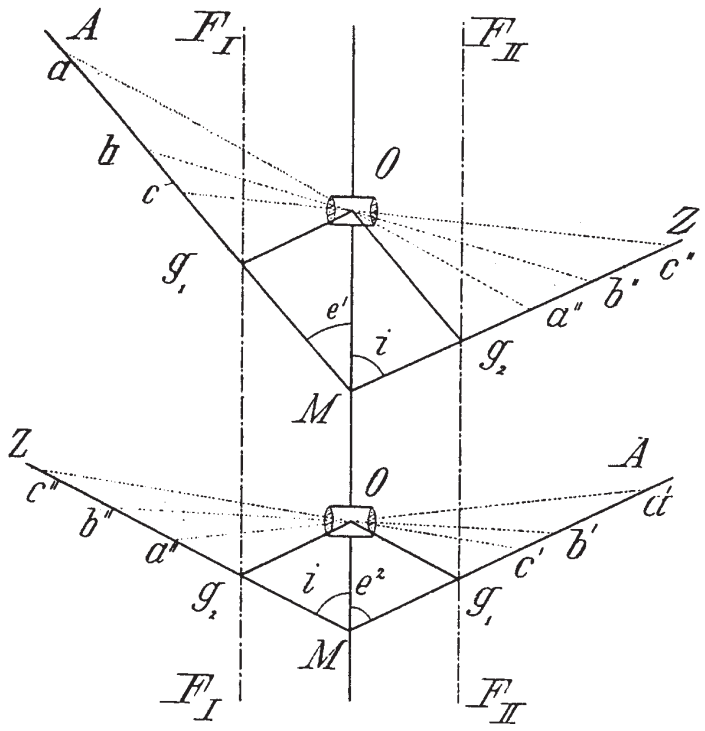
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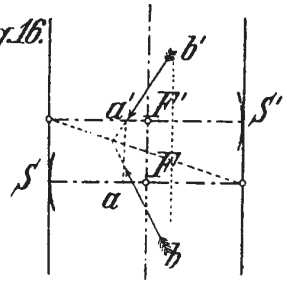
F'II

Fig.16.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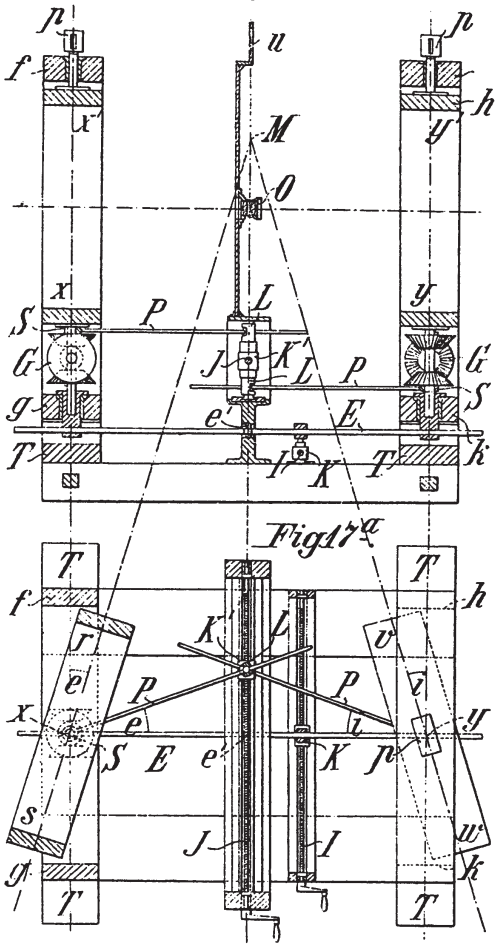
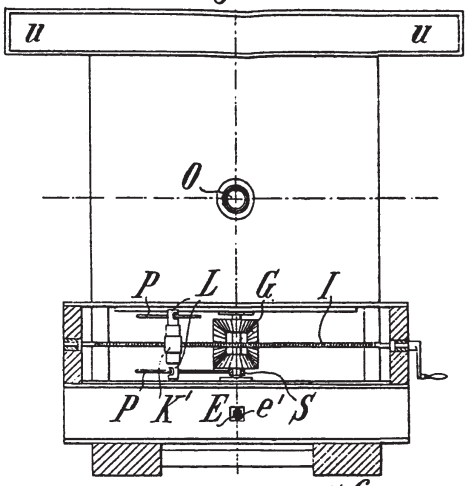


Fig17a.

Fig.17b.



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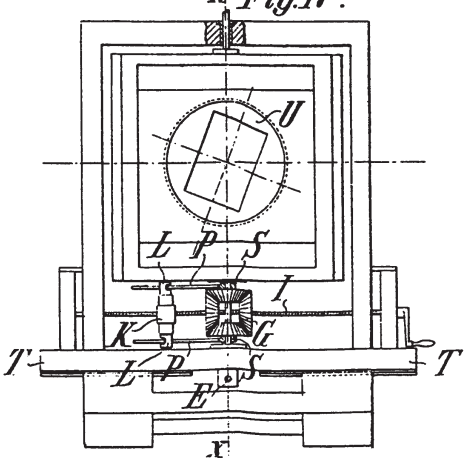

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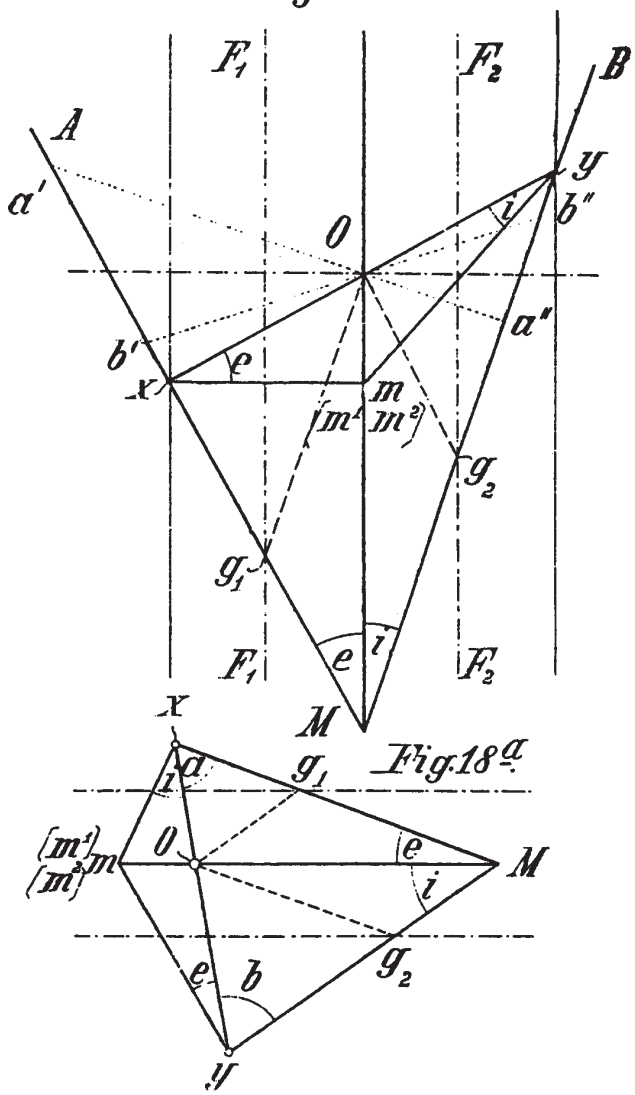


Fig.18a.

Fig.19.

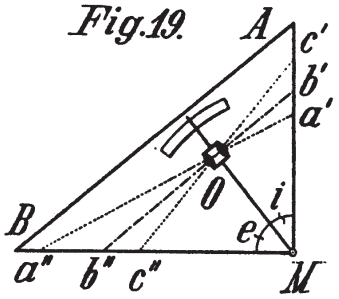


Fig.19a.

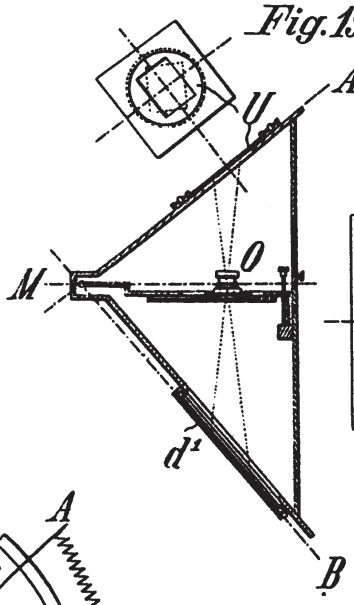
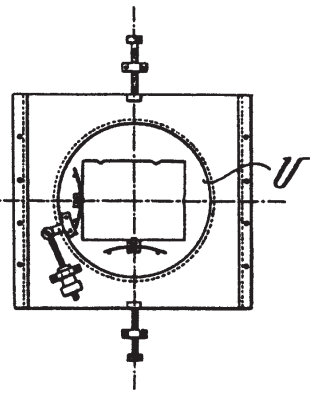


Fig.19b.



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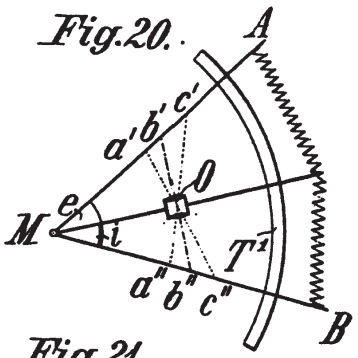

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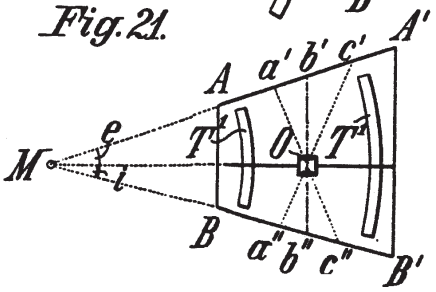

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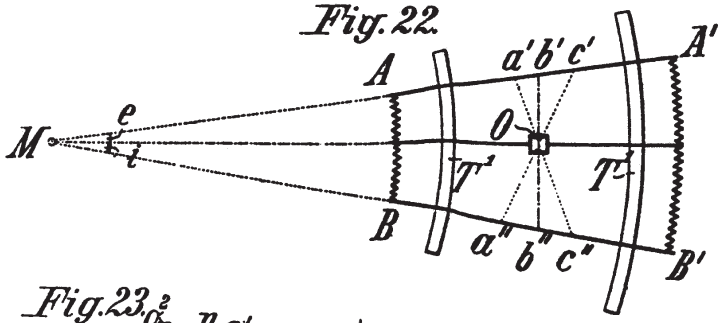

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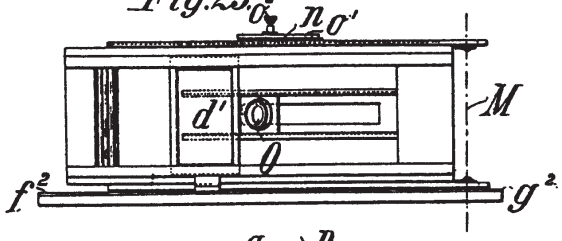

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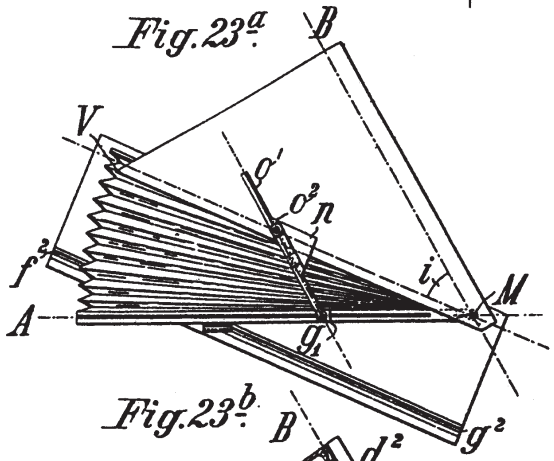
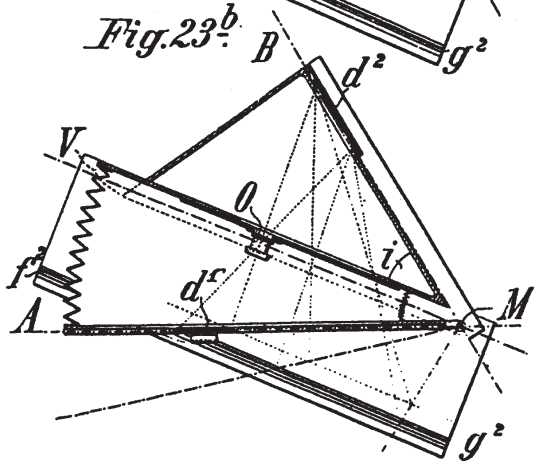


Fig. 23ᵇ.



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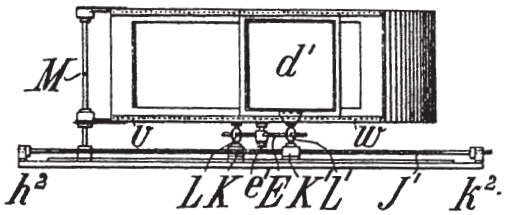

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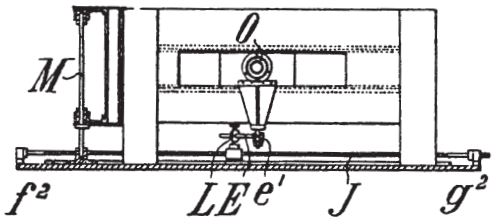
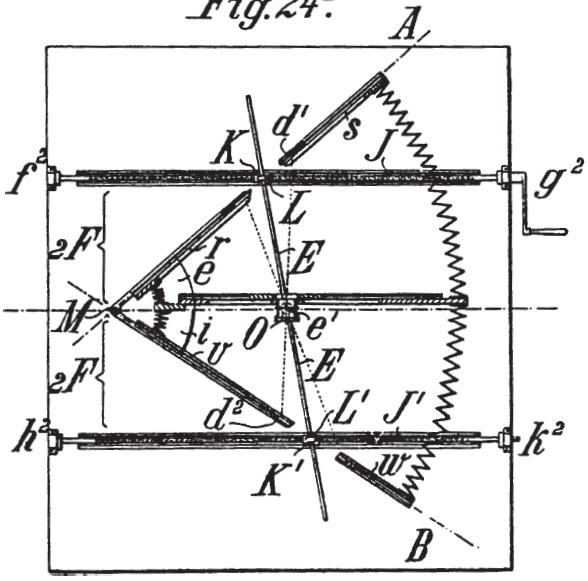


Fig.24ᵇ.



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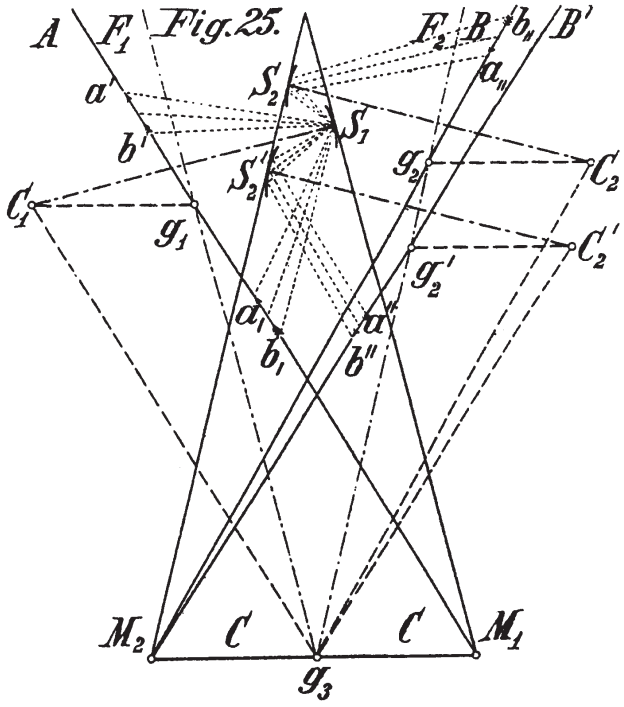
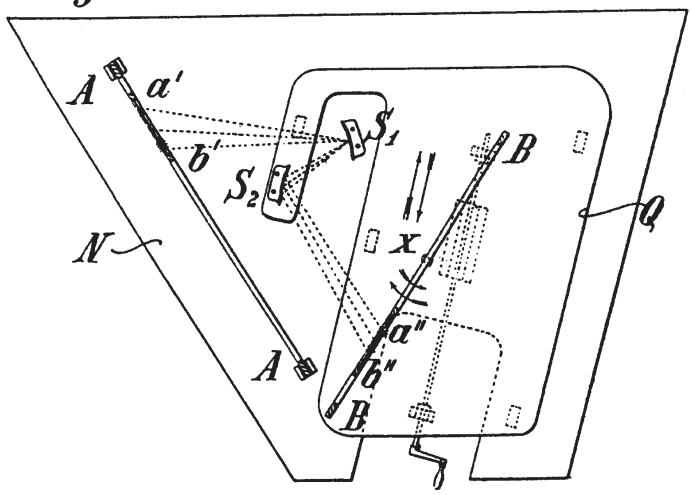


Fig.25a



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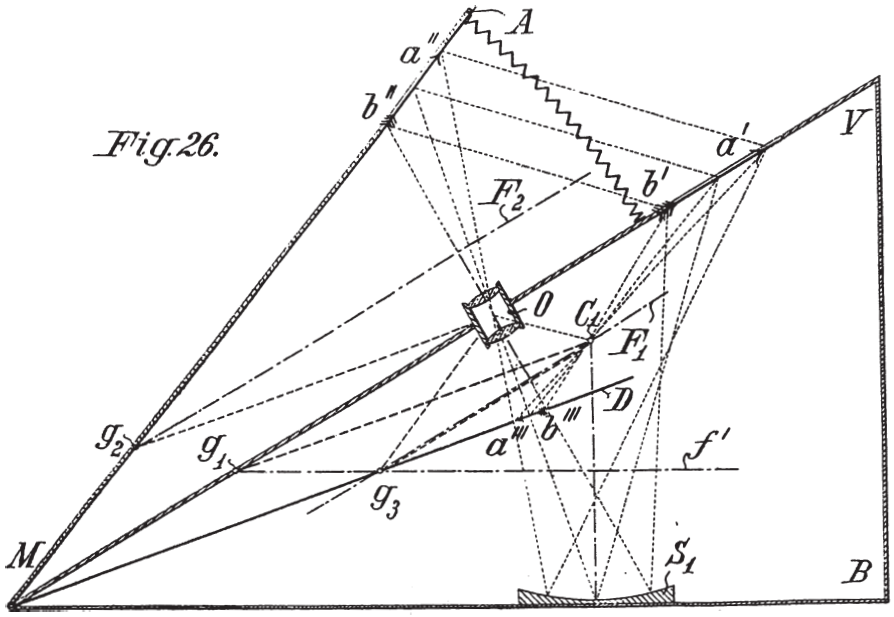

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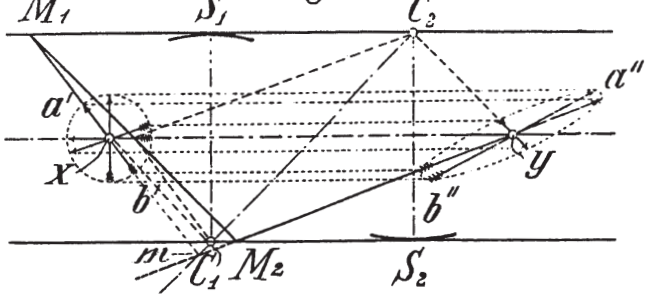


Fig. 27a.

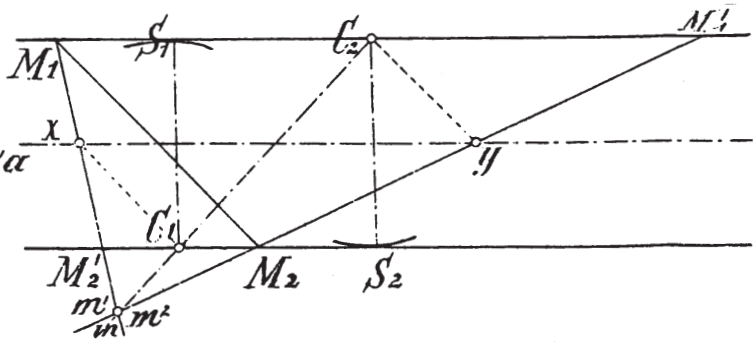
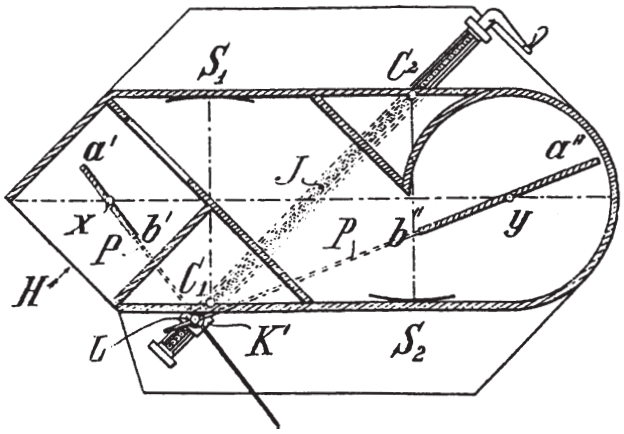


Fig. 27b.



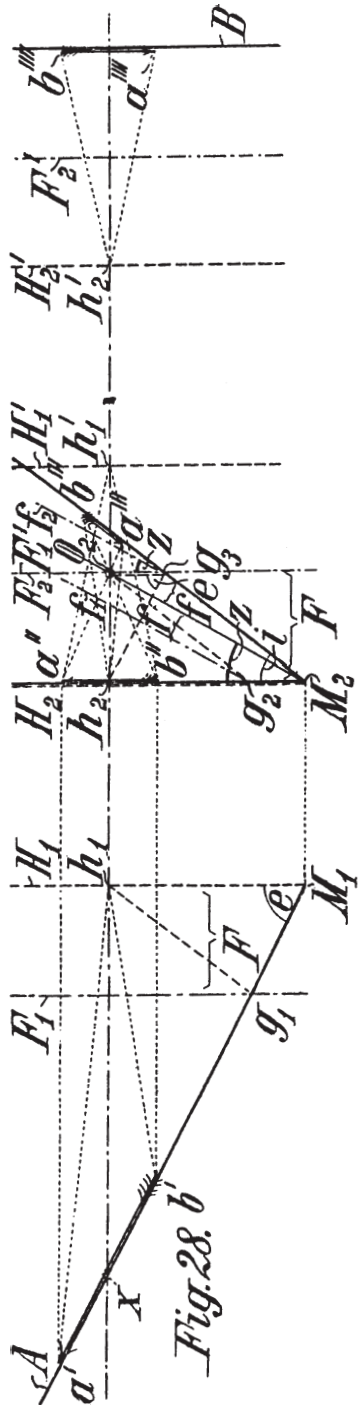


Fig. 28. b

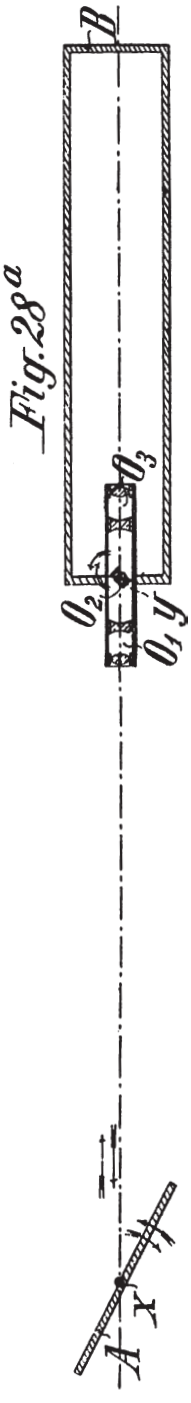


Fig. 28. a